Parameterization of Finite-Amplitude Waves in Climate Models

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Momentum Transport and Deposition

- When waves break they deposit momentum to the mean flow.
- Even before they break, there is a wave-induced mean flow associated with the waves
  - for surface waves, this is the "Stokes drift" . . . a.k.a. the wave-induced mean flow

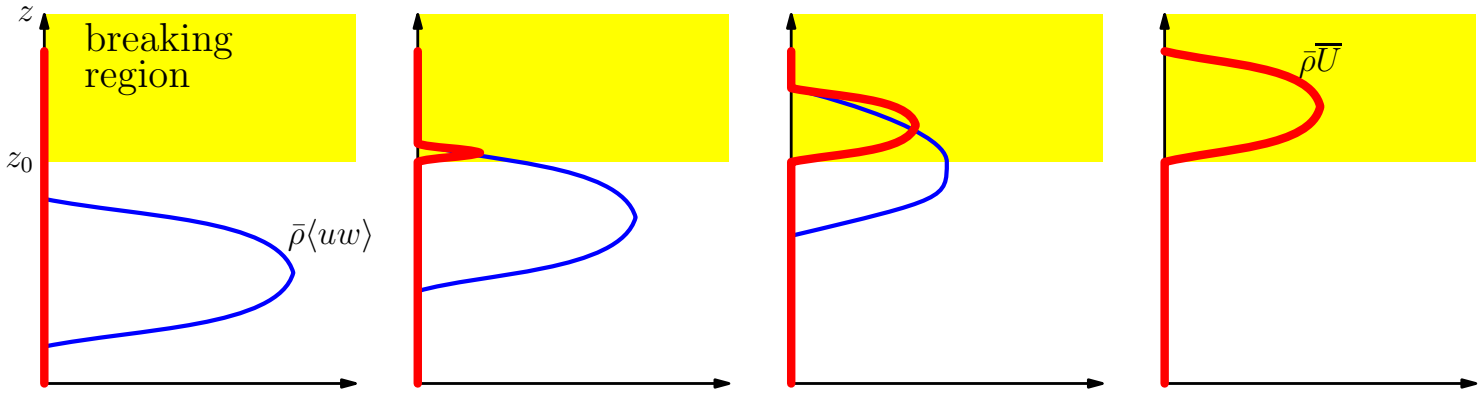




- Like surface waves, internal waves induce a mean flow
- Unlike surface waves, the wave-induced mean flow of internal waves has the most important influence upon the evolution of finite-amplitude wavepackets

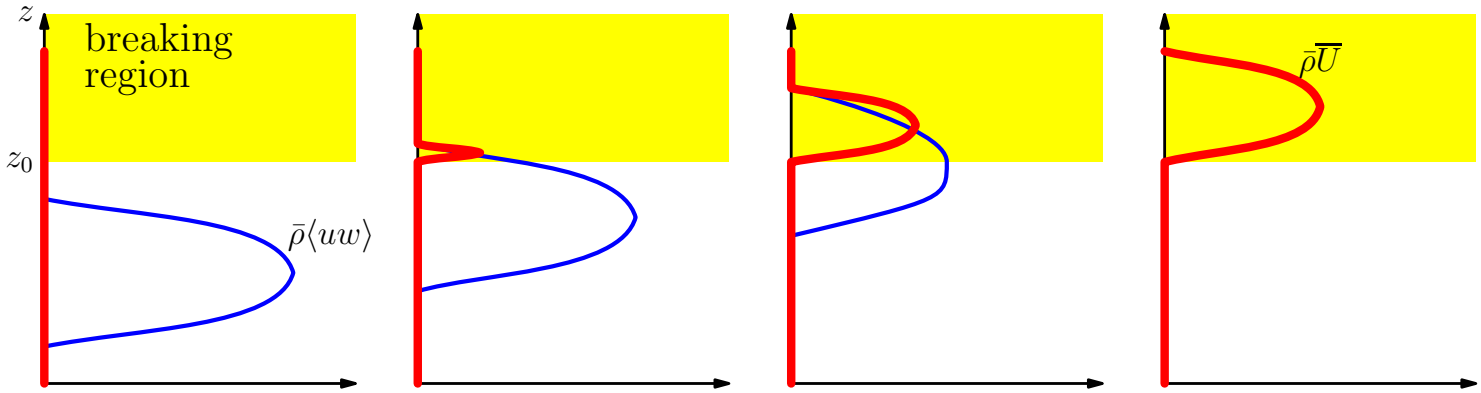
"Irreversible" and "Reversible" Momentum

- When waves break, momentum transported by the flux $\bar{\rho}\langle uw\rangle$ *irreversibly* accelerates the background according to $\partial_t \overline{U} = -\frac{1}{\bar{\rho}}\partial_z(\bar{\rho}\langle uw\rangle)$

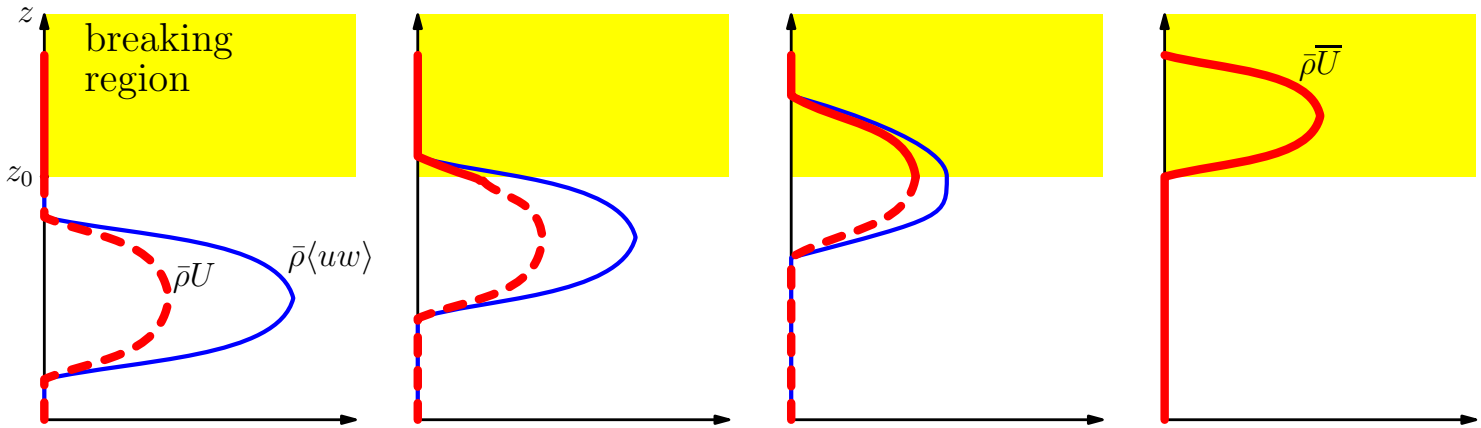


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- Beforehand they induce a *reversible* mean flow U such that $\partial_t U = -\frac{1}{\bar{\rho}}\partial_z(\bar{\rho}\langle uw\rangle)$
  - explicitly, $U = \langle uw\rangle / c_{gz} = -\langle \xi\zeta\rangle$ (ξ - vertical displacement; ζ - vorticity)
  - wave breaking irreversibly transforms this into a steady background flow: $U \to \overline{U}$.



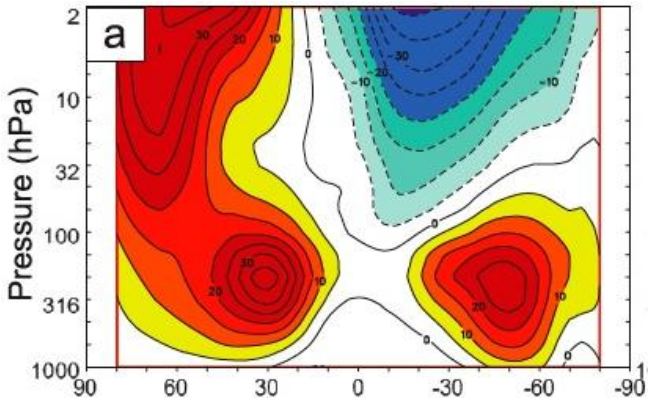
A Gravity Wave Drag Parameterization Scheme

- The Canadian Middle Atmosphere Model (CMAM) orographic GWD scheme:
  - the amplitude, A, of waves at the ground is set by hill height
  - $\mathsf{F} \equiv NA/\bar{U}$ is computed at successive heights
  - if $\mathsf{F} > \mathsf{F_c}$ at some height, the amplitude is reduced so that $\mathsf{F} = \mathsf{F_c}$
  - the corresponding momentum is deposited to the mean flow at that height

OBSERVED CLIMATOLOGY:

DJF zonal winds
(Northern Hemisphere Winter)

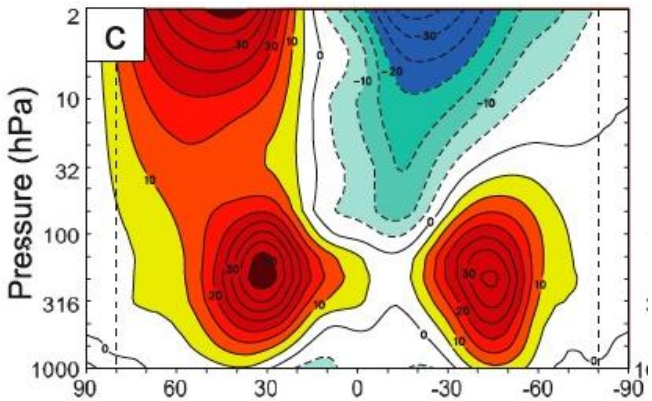
Using data from Randel et al (2004)



CMAM SIMULATIONS:
($\mathsf{F_c} = 0.71$)

DJF zonal winds

Figures from Scinocca & BRS (2010)



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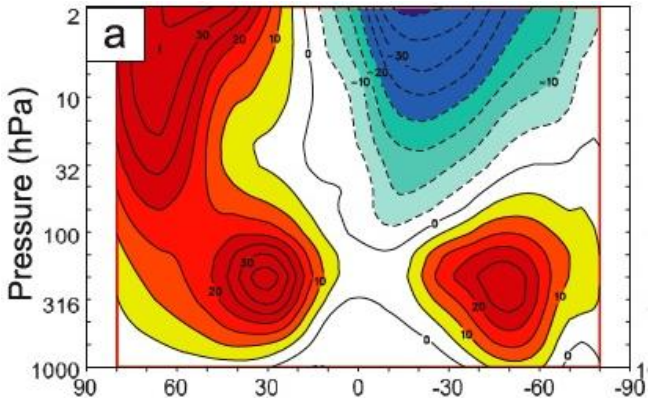
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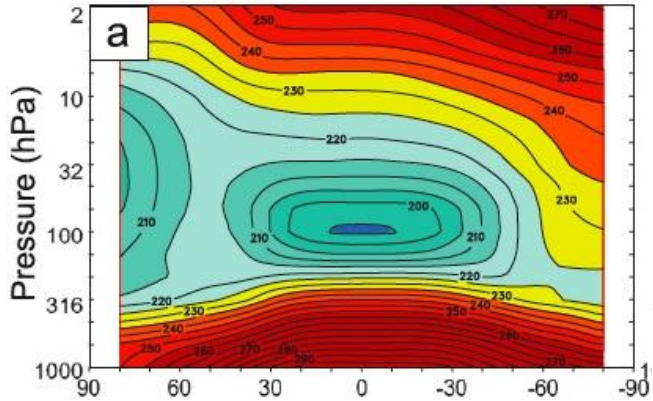
OBSERVED CLIMATOLOGY:

DJF zonal winds
(Northern Hemisphere Winter)

and **DJF temperature**

Using data from Randel et al (2004)



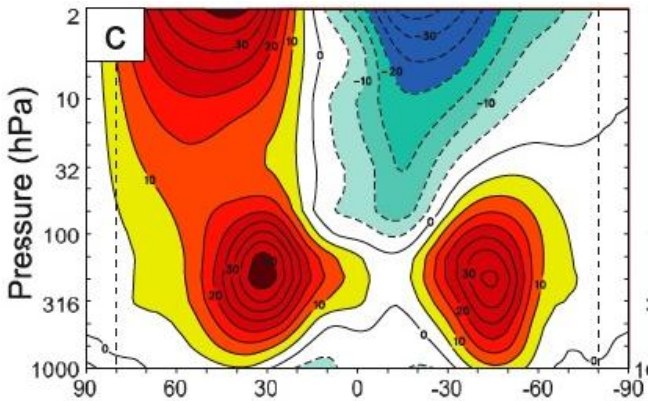


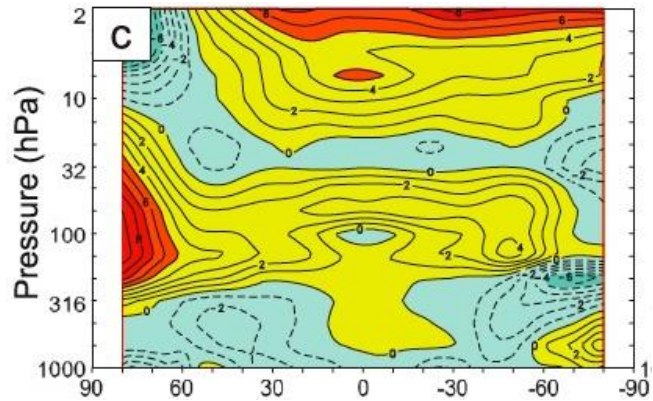
CMAM SIMULATIONS:
($\mathsf{F_c} = 0.71$)

DJF zonal winds

and **DJF temperature anomaly**
Note: *stronger shear over northern hemisphere jet stream gives warmer stratosphere over north pole due to thermal wind balance.*

Figures from Scinocca & BRS (2010)

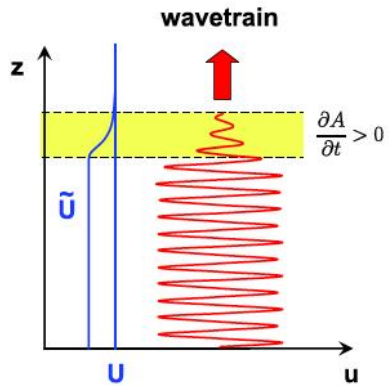




Modified Parameterization Scheme

- Assume the waves are affected not only by the background wind but also by the wave-induced mean flow (ie include "self-acceleration")

$$\langle u \rangle_L = -\tfrac{1}{2}\tfrac{N^2}{\Omega} k A^2 \quad \text{with} \quad \Omega = k\bar{U}.$$



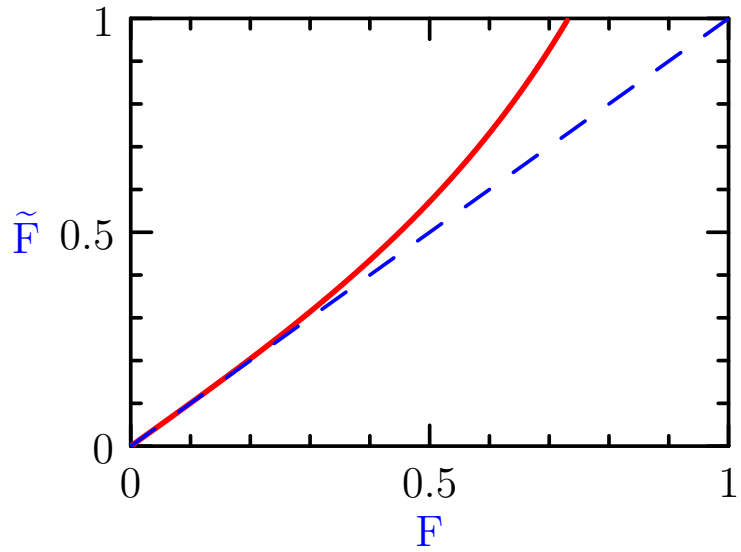
- So total horizontal flow is

$$\bar{U} + \langle u \rangle_L = \bar{U}\,(1 - \mathsf{F}^2/2)$$

in which $\mathsf{F} \equiv NA/\bar{U}$, as before.

- So the appropriate nonlinearity parameter characterizing stability is

$$\widetilde{\mathsf{F}} \equiv \frac{NA}{\bar{U} + \langle u \rangle_L} = \frac{\mathsf{F}}{1 - \frac{1}{2}\mathsf{F}^2}$$

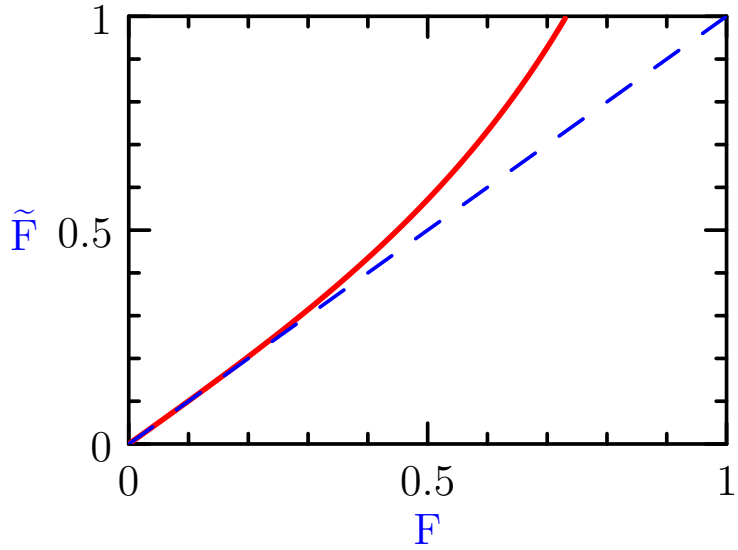


Modified Parameterization Scheme

- We propose that the criterion for breaking should be based upon a critical value, $\widetilde{\mathsf{F}}_c$, of

$$\widetilde{\mathsf{F}} = \frac{NA}{U_{\text{total}}} = \frac{\mathsf{F}}{1 - \frac{1}{2}\mathsf{F}^2}$$

- With no additional parameterization ...
  - the overturning condition is $\widetilde{\mathsf{F}}_c = 1$
  - so the corresponding unadjusted critical nonlinearity parameter is $\mathsf{F}_c = \sqrt{3} - 1 \simeq 0.73$.
  - this is close to the default value used by CMAM: $\mathsf{F}_c = 1/\sqrt{2} \simeq 0.71$



- In comparison with the present-day climate, we find that using a smaller value in simulations of $\widetilde{\mathsf{F}}_c \simeq 0.41 \Rightarrow \mathsf{F}_c = 0.38$ better captures observed temperature in middle atmosphere

Weakly Nonlinear Wavepacket Evolution

- For a quasi-monochromatic wavepacket, the mass-streamfunction amplitude envelope, $A_\Psi(z,t)$, changes in time due to ...
  - linear dispersion: $\propto (\partial_{mm}\omega)\,\partial_{zz}A_\Psi$
  - advection by the mean flow: $-U\partial_x\Psi \;\rightarrow\; -U\,\imath k A_\Psi$

- Taking U to be the wave-induced mean flow alone,

$$U = -\langle \xi\zeta \rangle \propto |A_\Psi|^2,$$

this gives the Nonlinear Schrödinger Equation in terms of A_Ψ:

$$\partial_t A_\Psi = \imath \tfrac{1}{2}\omega_{mm}\partial_{zz}A_\Psi - \imath k U A_\Psi.$$

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- The result is rigorously extended by perturbation theory.
  Including next-order terms gives

$$\partial_t A_\Psi = \imath\tfrac{1}{2}\omega_{mm}\partial_{zz}A_\Psi - \imath k U A_\Psi + \tfrac{1}{6}\omega_{mmm}\partial_{zzz}A_\Psi + \tfrac{1}{2}\frac{\omega^2}{N^2 k}\left(3m - \frac{\imath}{H_\rho}\right)(\partial_z U)A_\Psi$$

in which H_ρ is the density scale height.

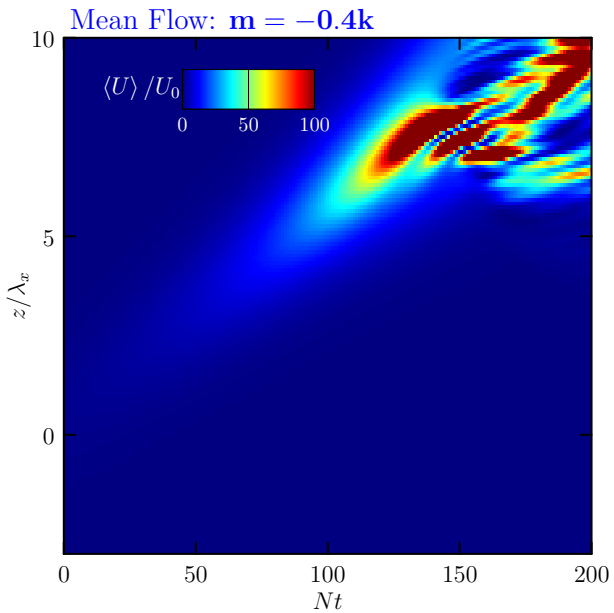
Solution of NLS equation

- Prescribe initial conditions so that the vertical displacement field is

$$\xi = A_{\xi 0}\, e^{-z^2/2\sigma^2} \left(e^{\imath(kx+mz-\omega t)}\, e^{z/2H_\rho} \right)$$

  in which $m = -0.4k$, $A_{\xi 0} \simeq 0.008\lambda_x$, $\sigma = H_\rho \simeq 1.6\lambda_x$

- Solution of NLS equation correctly shows initial exponential increase in amplitude of the wavepacket as it moves upward at the group velocity.

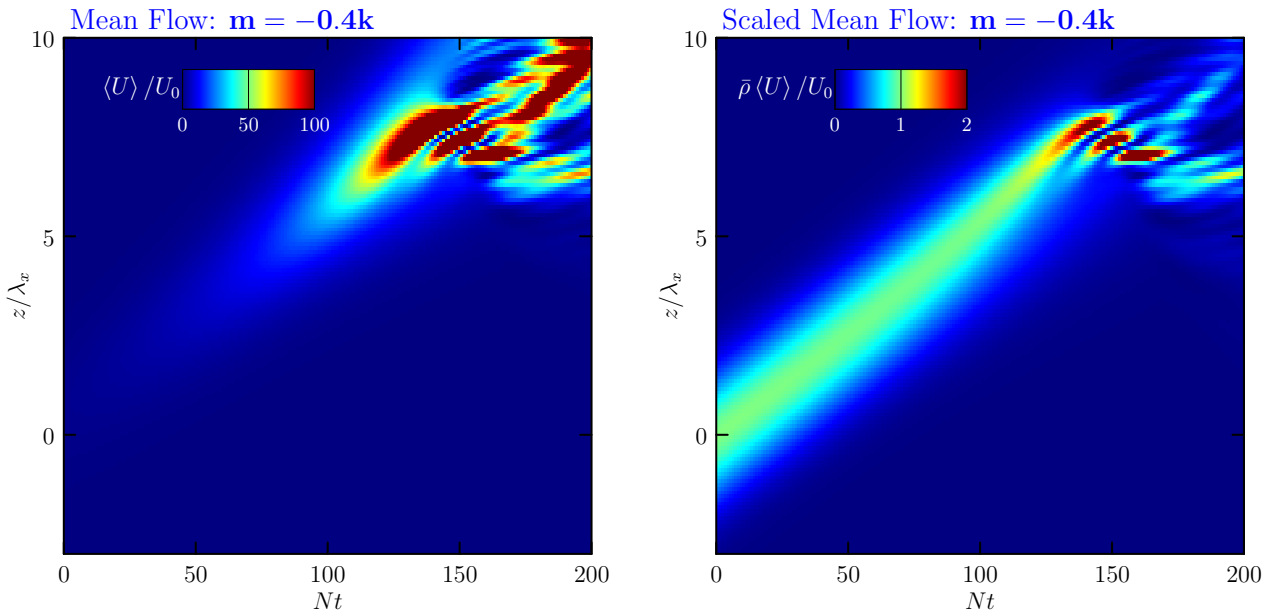


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- This can be scaled by $\bar{\rho}$ ...

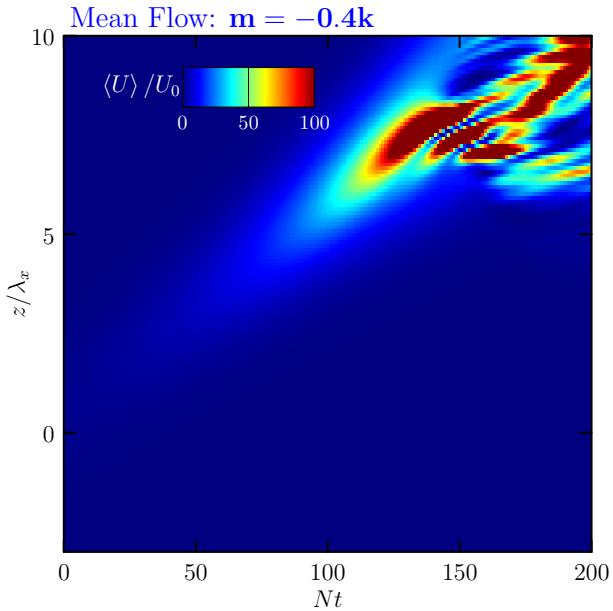


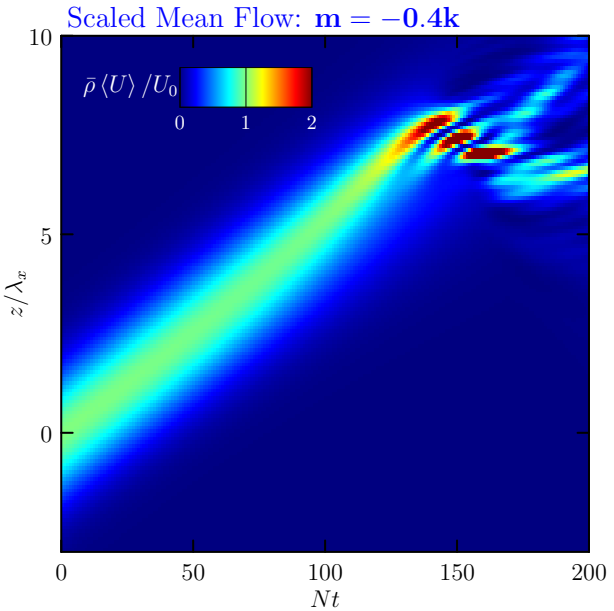
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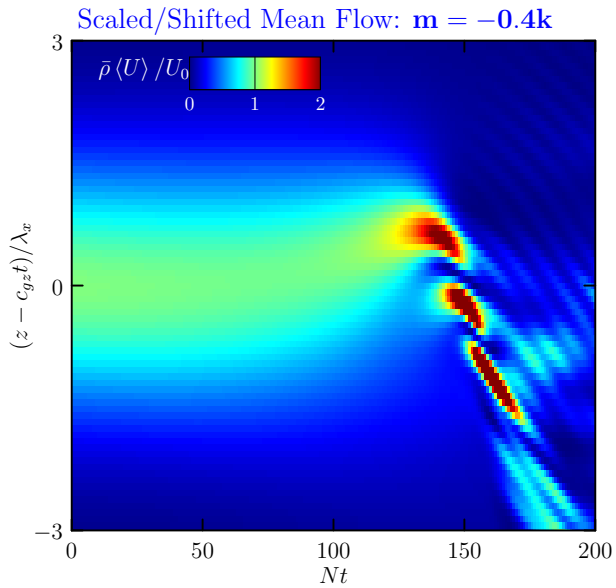
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- This can be scaled by $\bar{\rho}$ . . .
  and the z-axis shifted by $c_{gz}t$ in order to focus upon weakly nonlinear behaviour

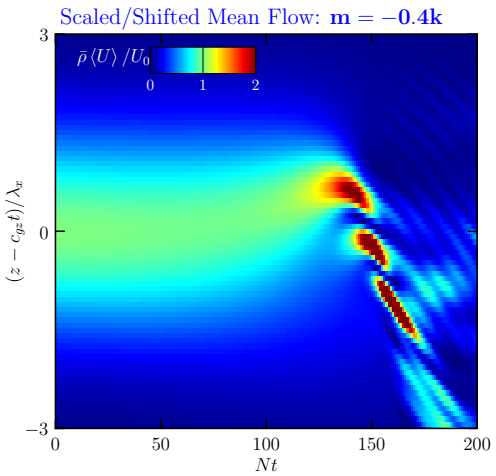


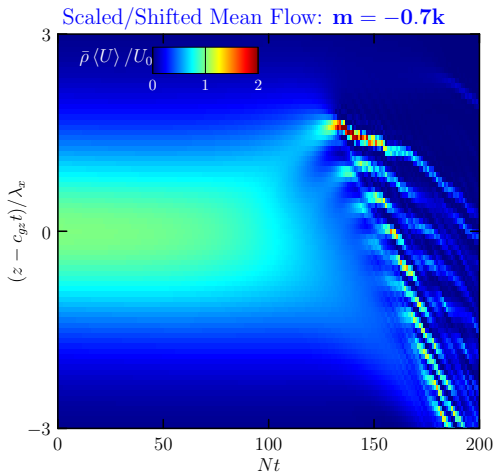


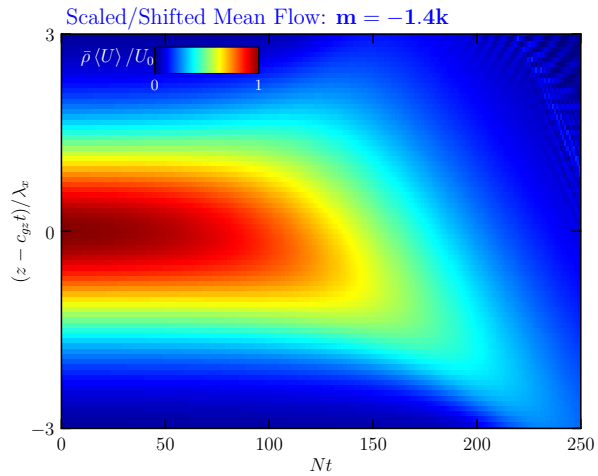


Modulational Stability and Instability

- Wavepackets are modulationally unstable if $|m| < |k|/\sqrt{2} \simeq 0.7k$ and are modulationally stable otherwise.
- Weakly nonlinear wavepackets which are modulationally unstable (eg $m = -0.4k$ below) narrow as their amplitude grows at a faster than the anelastic growth rate.
- Weakly nonlinear wavepackets which are modulationally stable (eg $m = -0.4k$ below) broaden as their amplitude grows less quickly than the anelastic growth rate.





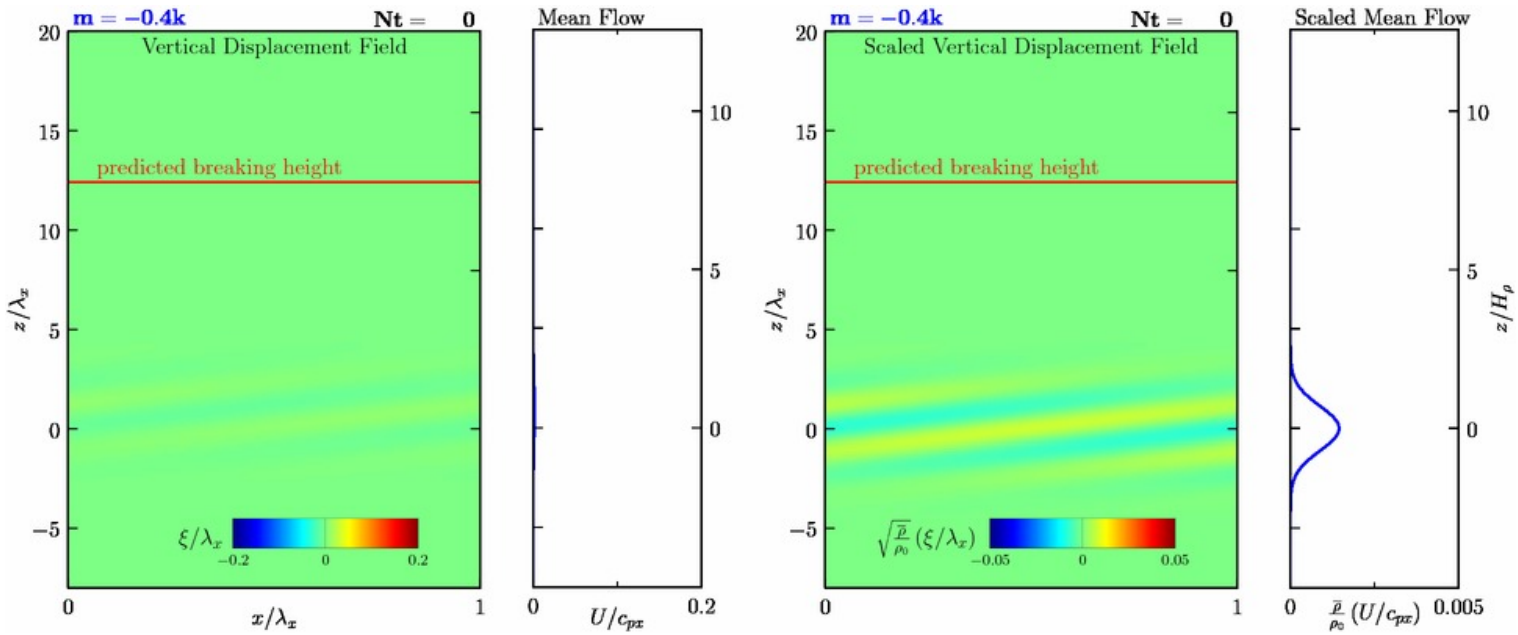


A Modulationally Unstable Wavepacket

- Wavepackets with $|m| < |k|/\sqrt{2} \sim 0.71|k|$ are modulationally unstable.
- The interaction becomes more significant as the waves grow in amplitude
- Eventually this changes the wave structure: the packet narrows and increases in amplitude
- This leads to overturning below the level predicted by linear theory

Simulation initialized with wavepacket: $\xi = A_{\xi 0}\, e^{-z^2/2\sigma^2} \left(e^{\imath(kx+mz-\omega t)} e^{z/2H_\rho} \right)$

$$m = -0.4k, \quad A_{\xi 0} \simeq 0.008\lambda_x, \quad \sigma = H_\rho \simeq 1.6\lambda_x$$

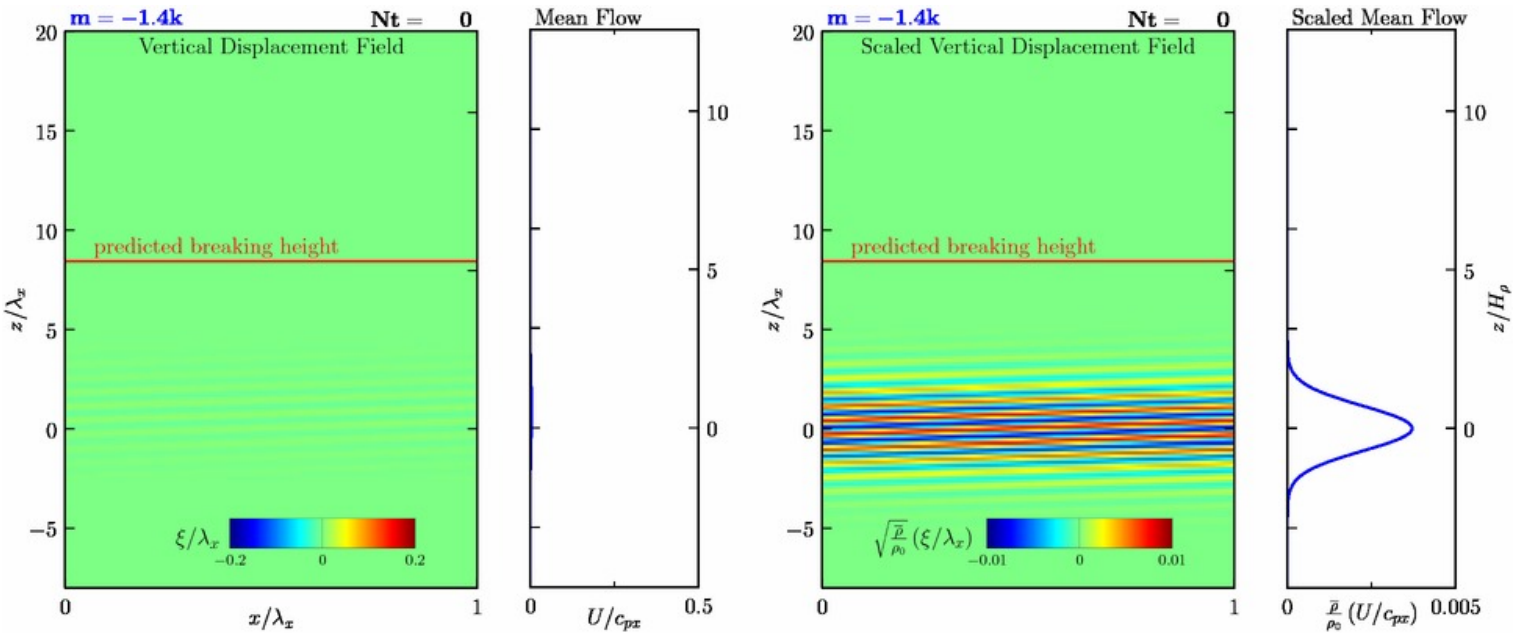


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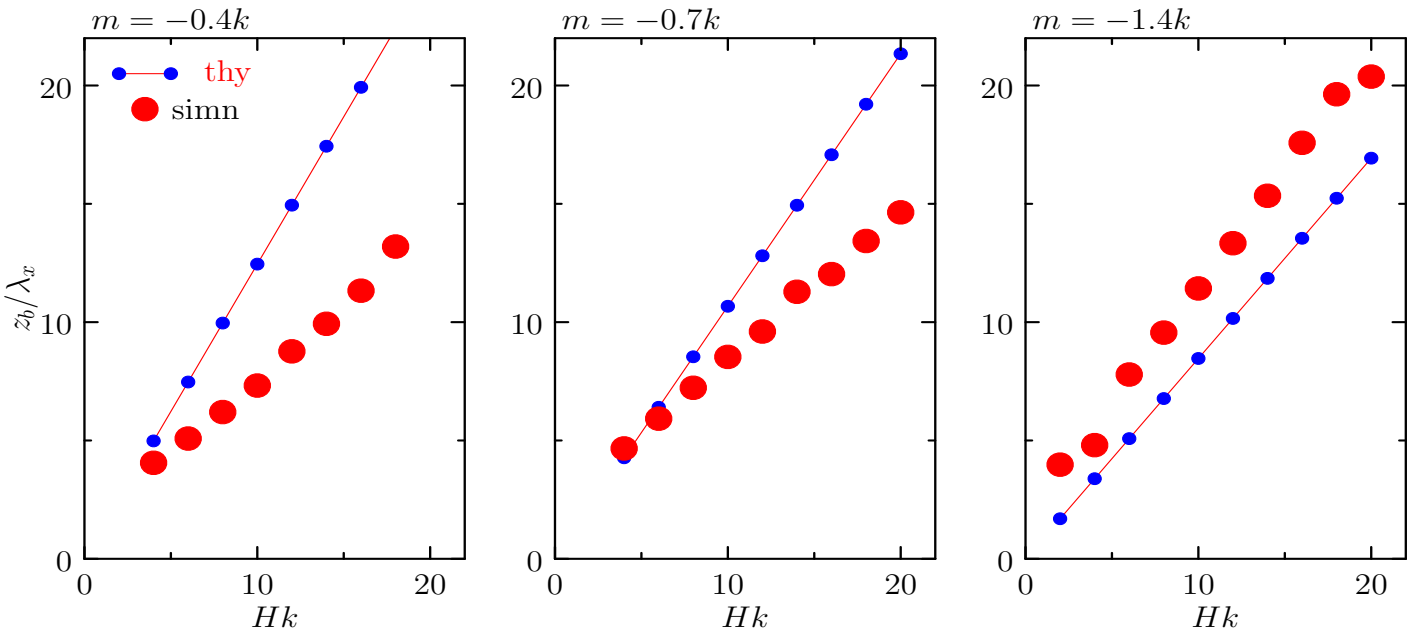
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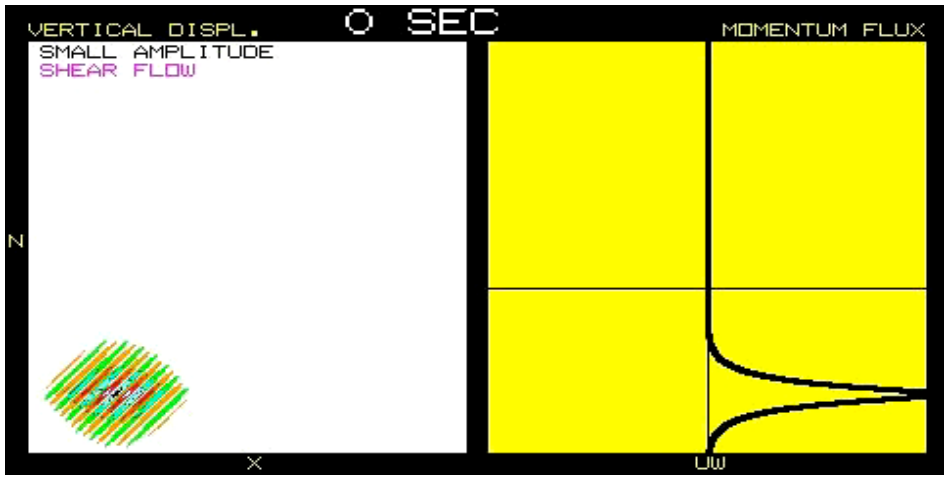
Simulated Overturning Heights

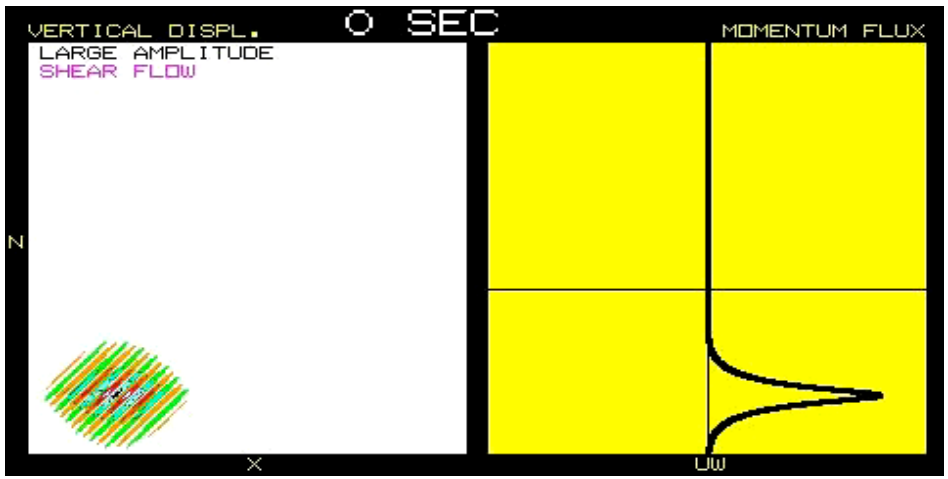
- For strongly non-hydrostatic waves with $|m| \lesssim |k|/\sqrt{2} \simeq 0.71|k|$ waves break below the level predicted by linear theory. Breaking occurs well below if $Hk \gg 1$.
- For more hydrostatic waves with $|m| \gtrsim |k|/\sqrt{2}$ waves break above the level predicted by linear theory. Breaking occurs well above if $|m/k|$ is large.



Boussinesq Wavepacket Propagation in Opposing Shear

- With realistic winds and stratification, other deviations from linear theory occur . . .
  small-amplitude waves reflect; large-amplitude waves propagate above reflection level





[BRS, QJRMS (2000)]

Discussion

- In the process of "self-acceleration", internal waves induce a mean flow that interacts significantly with the waves if their amplitude is sufficiently large ($A_\xi \gtrsim 0.02\lambda_x$).

- Low-frequency waves ($\omega \lesssim 0.8N$) break above the level predicted by linear theory.
  - Relative breaking height increases as $|m/k|$ increases.

- Although horizontal periodicity is necessary for mean flow, the same qualitative results occur for horizontally compact wavepackets
  - in this case need to superimpose $-\xi\zeta$ pointwise on the initial horizontal velocity field
  - as the packet evolves the mean flow over a wavelength acts locally to Doppler-shift the wavepacket

- Details of 3D wave breaking are being examined (in collaboration with Tom Lund, CoRA)

