The application of wave-activity conservation laws to cloud resolving model simulations of multiscale tropical convection

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What is a wave-activity conservation law?

- A wave-activity conservation law is a relation of the form:

$$\frac{\partial A}{\partial t} + \nabla \cdot \mathbf{F} = 0 \qquad \begin{array}{l} A = \text{wave-activity density} \\ \mathbf{F} = \text{wave-activity flux} \end{array}$$

  $\rightarrow$ A is a quantity that is conserved in the absence of forcing and dissipation (in contrast to disturbance energy and enstrophy).

- Useful in the study of disturbances ξ' (waves, eddies) to some background flow X i.e. $\xi = X + \xi'$ with $A \sim O(\xi'^2)$.

- In midlatitudes one takes $\xi = X(y,z,t) + \xi'(x,y,z,t)$ and defines $A^{\mathcal{M}_x} = \overline{q'^2}/2Q_y$, which satisfies the conservation law

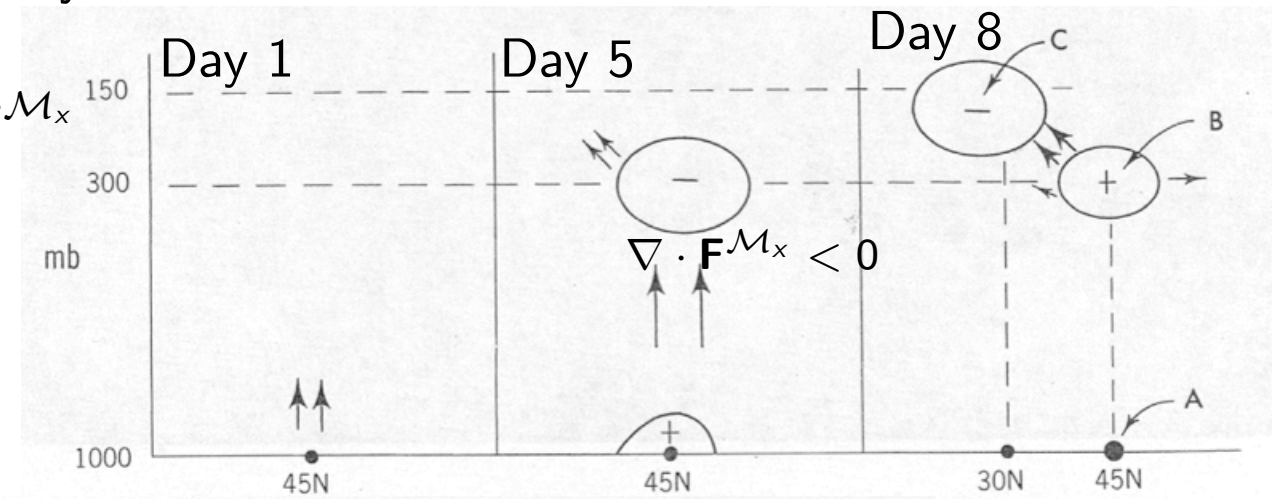
$$\frac{\partial A^{\mathcal{M}_x}}{\partial t} + \underbrace{\nabla \cdot \mathbf{F}}_{\text{Eliassen-Palm flux divergence}} = S \quad \text{with} \quad \mathbf{F}^{\mathcal{M}_x} = -\overline{u'v'}\,\hat{\mathbf{j}} + \frac{f_0}{N^2}\overline{v'b'}\,\hat{\mathbf{k}}; \quad S = \frac{1}{Q_y}\overline{S'q'}$$

  where Q and q' are the background and disturbance potential vorticity and S includes both sources and sinks S'.

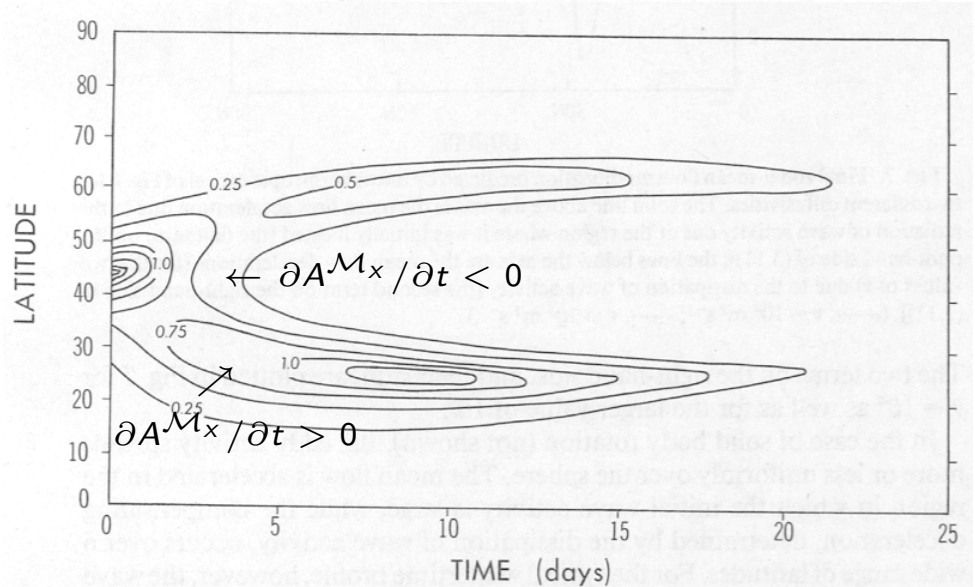
Application to large-scale circulations

- Evolution of pseudomomentum flux during a baroclinic life cycle:

$\nabla \cdot \mathbf{F}^{\mathcal{M}_x}$



$A^{\mathcal{M}_x}$



DAY 8

$$\nabla \cdot \mathbf{F}^{\mathcal{M}_x} = -\frac{\partial A^{\mathcal{M}_x}}{\partial t}$$

$\rightarrow$ Transience of pseudomomentum density explains flux divergence

Held & Hoskins (1985)

Application to the mesoscale

- On mesoscales one can consider disturbances to a vertically dependent shear-stratified flow.

  $\rightarrow$ Variables decomposed as $\xi = X(z) + \xi'(x, y, z, t)$.

  $\rightarrow$ Disturbance is the mesoscale convection and waves and the background is a shear-stratified flow.

- The 2D pseudomomentum $A^{\mathcal{M}_x}$ wave-activity conservation law for a $[\overline{u}(z), \overline{\theta}(z)]$ background state is (Scinocca & Shepherd 1992):

$$\frac{\partial A^{\mathcal{M}_x}}{\partial t} + \nabla \cdot \mathbf{F}^{\mathcal{M}_x} = S^{\mathcal{M}_x} \quad \text{where}$$

$$A^{\mathcal{M}_x} = \underbrace{\frac{\rho_0}{\overline{\theta}_z}\omega'\theta'}_{\text{gravity wave}} - \underbrace{\frac{1}{2}\frac{\rho_0^2}{(\overline{\theta}_z)^2}\left(\frac{\overline{u}_z}{\rho_0}\right)_z(\theta')^2}_{\text{vorticity wave}}; \qquad F^{\mathcal{M}_x}_{(z)} = \rho_0 u' w'$$

  in the small amplitude limit (can be generalized to finite amplitude).

  $\rightarrow$ Shaw & Shepherd (2008, 2009) derived the 3D conservation laws and showed how the conservation laws impact the resolved-scale flow in a climate model.

Application to mesoscale momentum transfer

- Pseudomomentum transfers involve two important mesoscale processes:
  - → Convective momentum transport (CMT); transport associated with tilted convective structures (Moncrieff 1992).
  - → Gravity wave momentum transport (GWMT); transport associated with vertically propagating gravity waves.
  - → Processes currently parameterized independently in climate models, which is inconsistent.
- Here we diagnose pseudomomentum wave-activity conservation law from the cloud resolving model simulations of Lane & Moncrieff (2008, 2010).
  - → Assess the cause of the vertical momentum flux convergence i.e. transience versus source/sink non-conservative term.

Cloud resolving model simulations

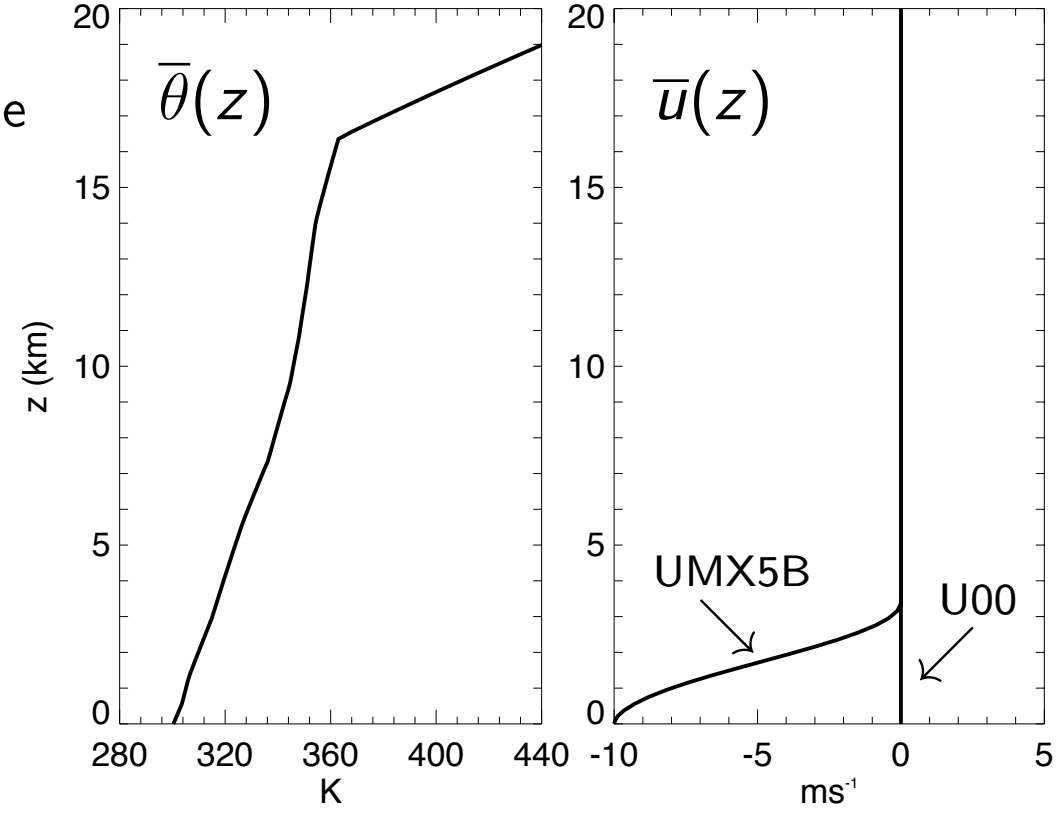
- Cloud resolving model is the anelastic model of Clark et al. (1996)

→ Two-dimensional domain: 2000 km wide and 40 km deep with Δx=1 km and Δz = 50 m to 200 m.

→ The imposed thermodynamic sounding was observed over the Tiwi Islands, Australia.

→ The imposed zonal wind is either zero (U00) or is westward below 3 km (UMX5B).

→ Simulations run for 180 hours

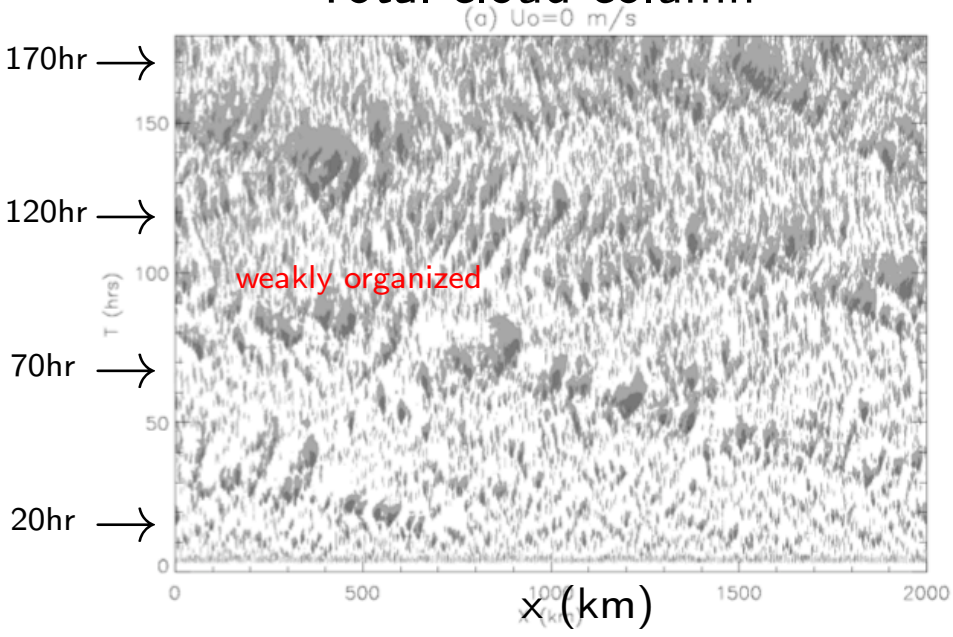


Momentum transfers in unsheared background

- In U00 simulation, shallow convection develops initially and eventually self-organizes into clusters which propagate at 5 m/s without a preferred direction

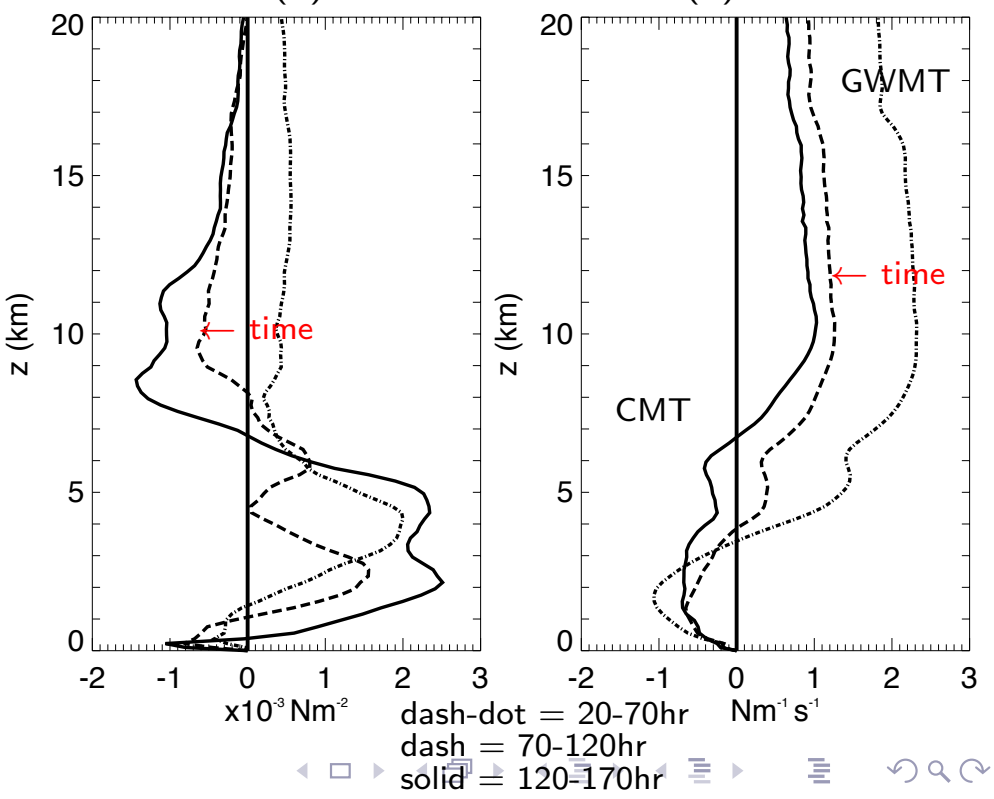
  $\rightarrow$ Convection is weakly organized.

Total cloud column



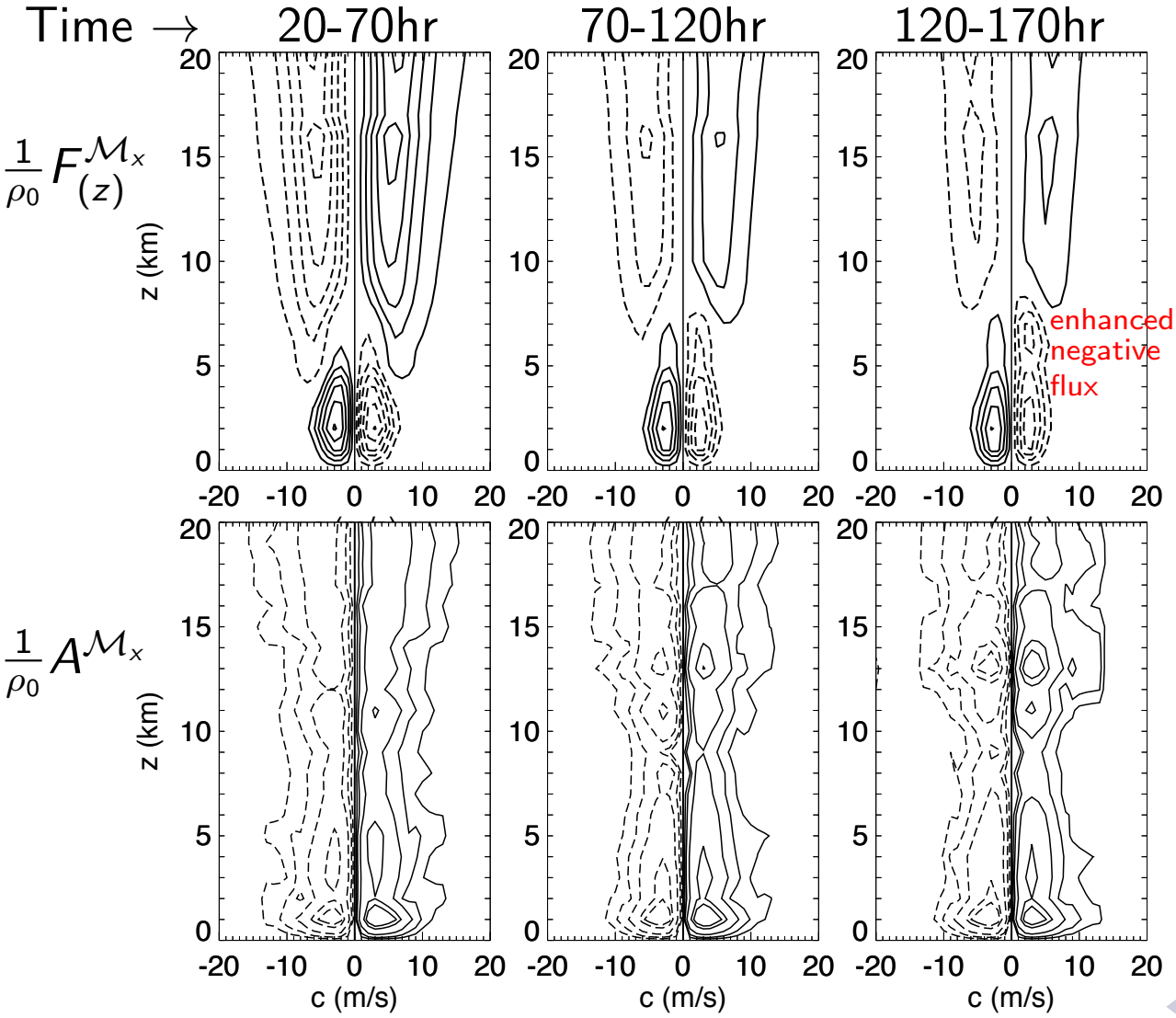
1st EP theorem: $\overline{p'w'} = \rho_0(U-c)F^{\mathcal{M}_x}_{(z)}$

$\overline{F^{\mathcal{M}_x}_{(z)}} = \overline{F^{\mathcal{E}}_{(z)}} = \overline{p'w'}$

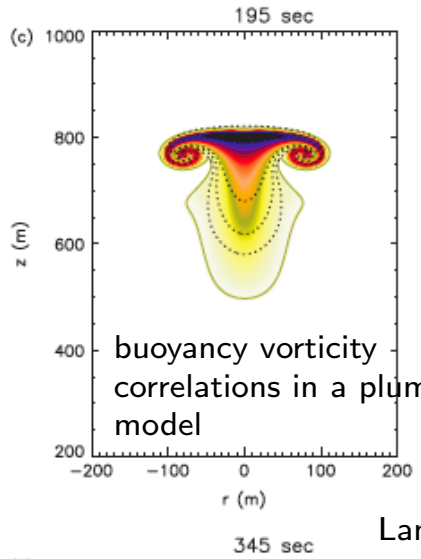


Momentum transfers in unsheared background

- Phase speed versus height co-spectra for U00:



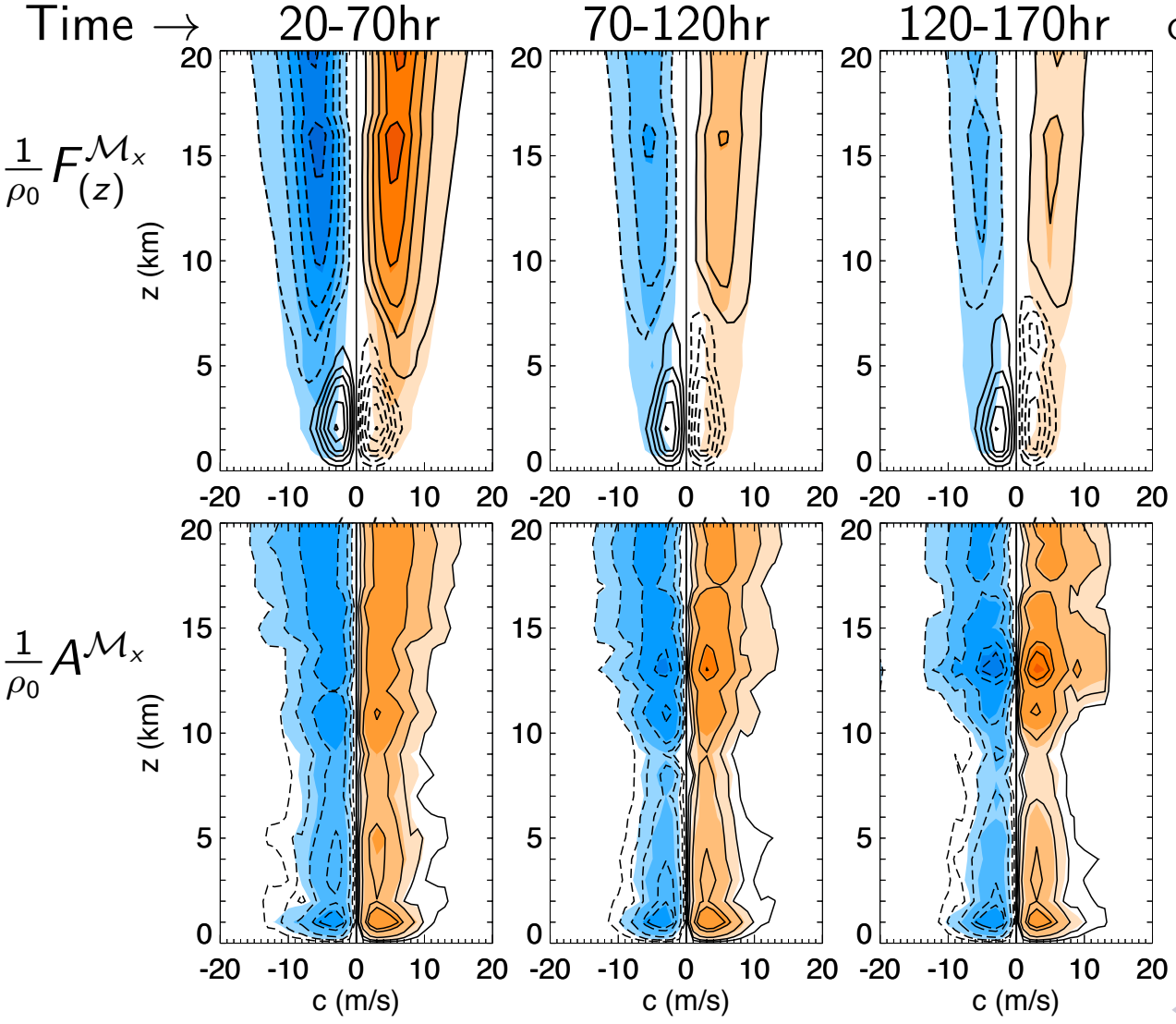
- ◇ Symmetric spectrum except between 120-170hr
- ◇ Flux changes sign above convectively active region
- ◇ Flux broadens with height
- ◇ Pseudomomentum density $A^{\mathcal{M}_x}$ changes between 10-15km and below 2km



Lane (2008)

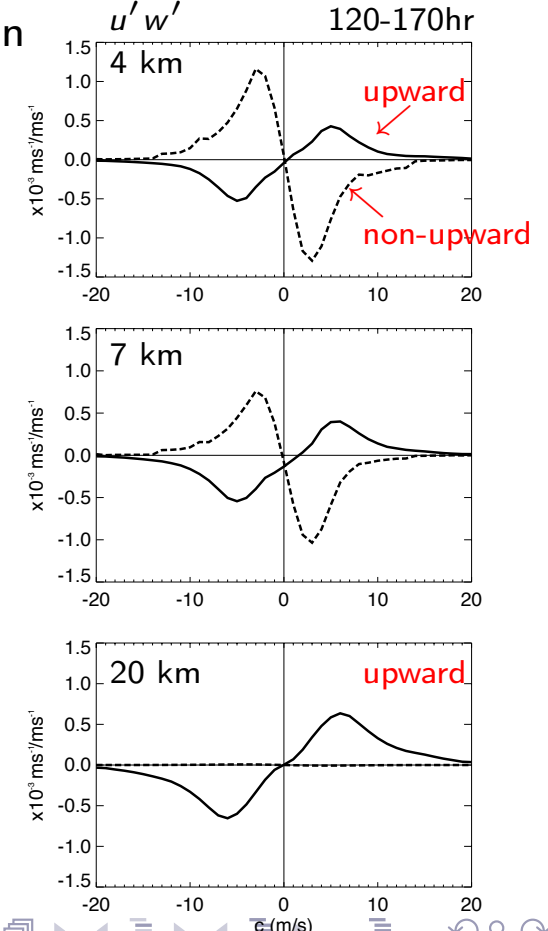
Momentum transfers in unsheared background

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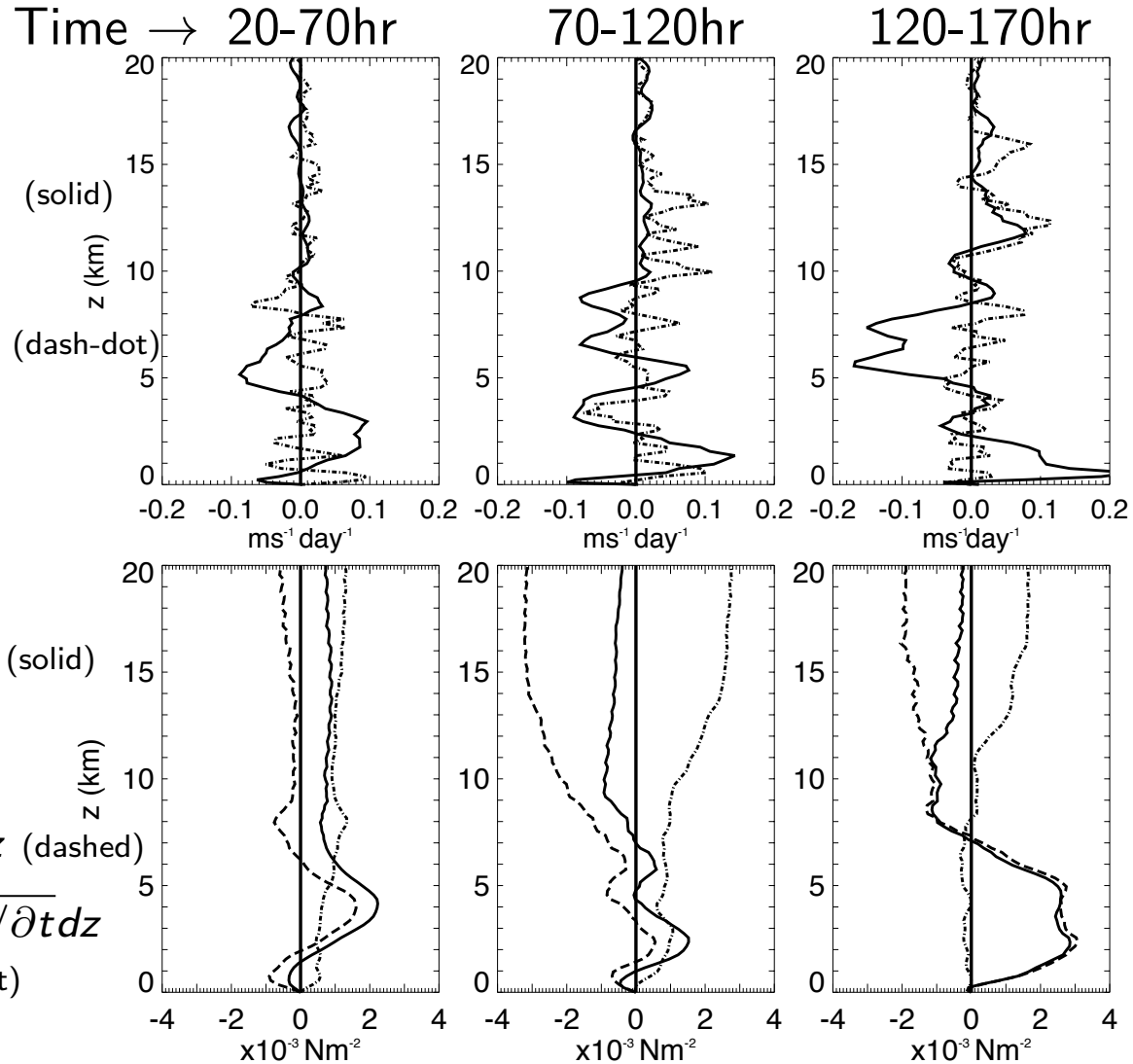
Colors indicate upward propagation

◇ Upward flux does not change sign below convectively active region



Momentum transfers in unsheared background

- Pseudomomentum flux convergence and its contributions



◇ Source/sink term dominates vertical transfer except between 12-14 km

◇ Divergence/convergence pattern below 10 km

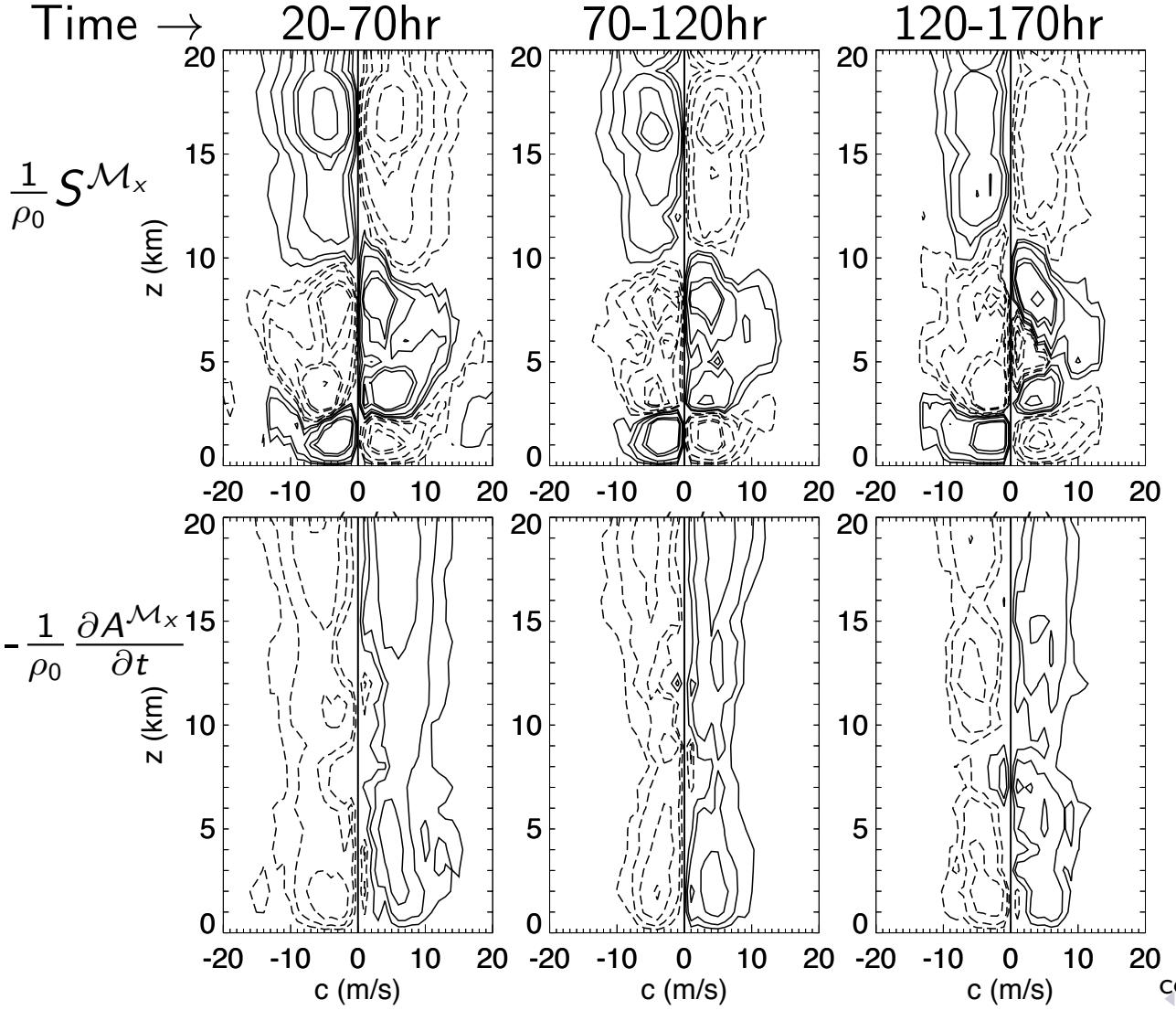
$$\frac{\partial \overline{F^{\mathcal{M}_x}_{(z)}}}{\partial z} = \overline{S^{\mathcal{M}_x}} - \overline{\frac{\partial A^{\mathcal{M}_x}}{\partial t}}$$

◇ Source/sink term dominates but there is cancellation

◇ Transient term usually > 0 and significant up to 20 km

Momentum transfers in unsheared background

- Phase speed versus height co-spectra for U00:



◇ Divergence/convergence pattern consistent with a source between 3-10km and a sink above 10km

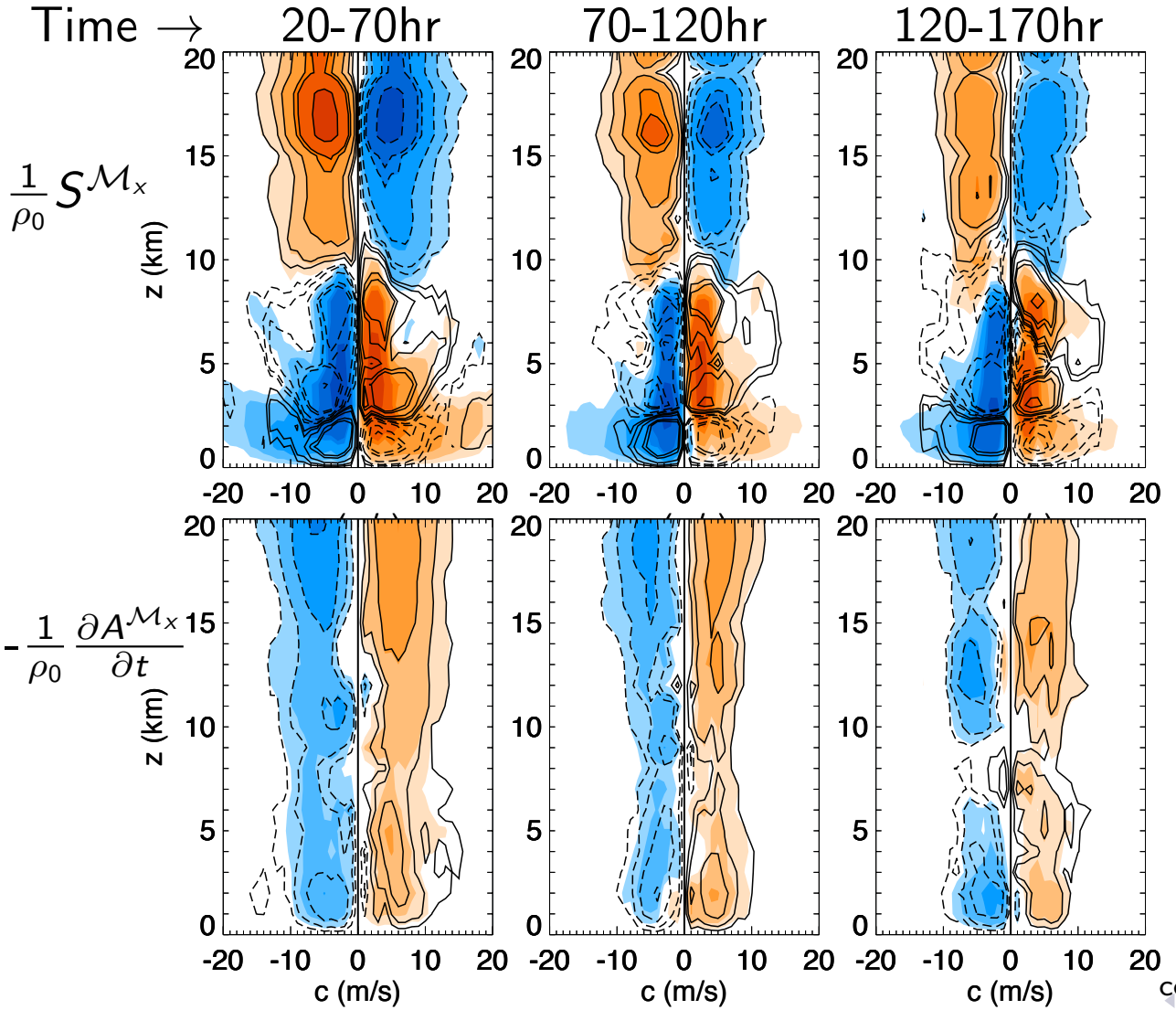
$$\frac{\partial \overline{F^{\mathcal{M}_x}_{(z)}}}{\partial z} = \overline{S^{\mathcal{M}_x}} - \overline{\frac{\partial A^{\mathcal{M}_x}}{\partial t}}$$

◇ Transience provides a source at all vertical levels

contour = 0.025 m/s/day

Momentum transfers in unsheared background

- Phase speed versus height co-spectra for U00:



- ◇ Divergence/convergence pattern consistent with a source between 3-10km and a sink above 10km
- ◇ The convergence for $c > 0$ below 5km is not consistent with upward propagation

$$\overline{\frac{\partial F^{\mathcal{M}_x}_{(z)}}{\partial z}} = \overline{S^{\mathcal{M}_x}} - \overline{\frac{\partial A^{\mathcal{M}_x}}{\partial t}}$$

- ◇ Transience provides a source at all vertical levels
- ◇ Transience contribution consistent with upward propagation

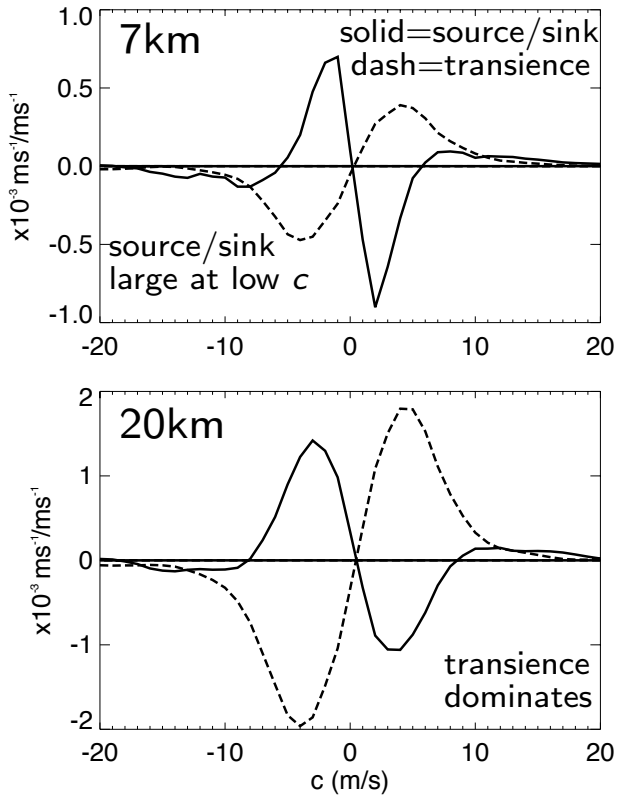
contour = 0.025 m/s/day

Momentum transfers in unsheared background

- Pseudomomentum flux convergence involves:
  - → CMT which peaks at ±2 m/s below 8 km.
  - → GWMT which peaks at ± 4-6 m/s from the surface up to 20 km.
- Upward propagating part of spectrum is consistent between the two regions.
  - → GWMT source is below 7 km.
- Transient signal near convective outflow from 10-14 km which peaks at -4 m/s.
- Source/sink dominates below 8 km.
- Transient signal dominates aloft.

Contribution to spectra

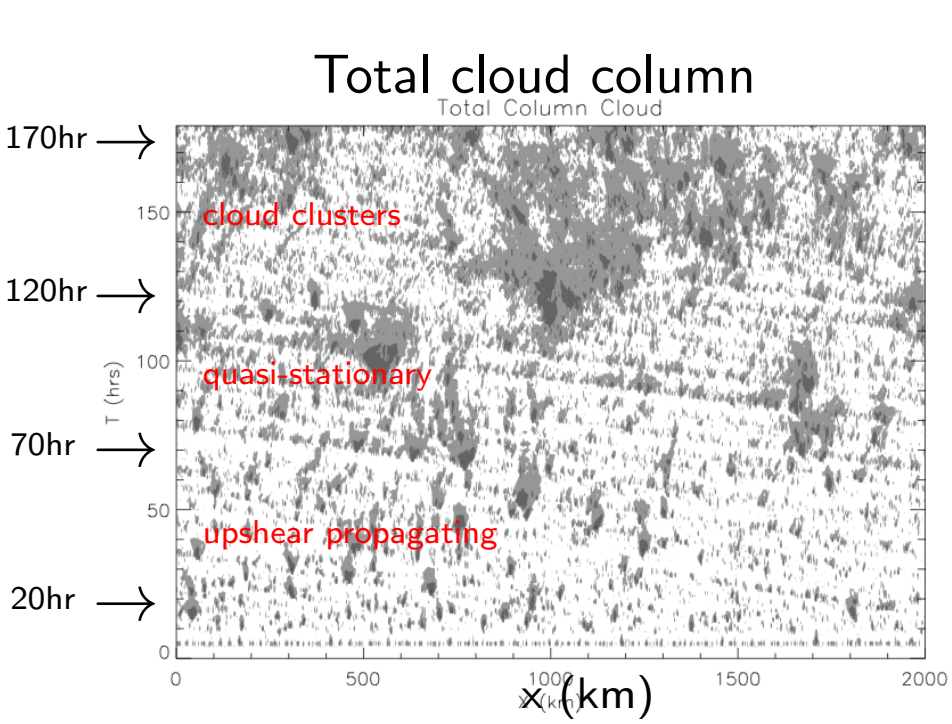
120-170hr

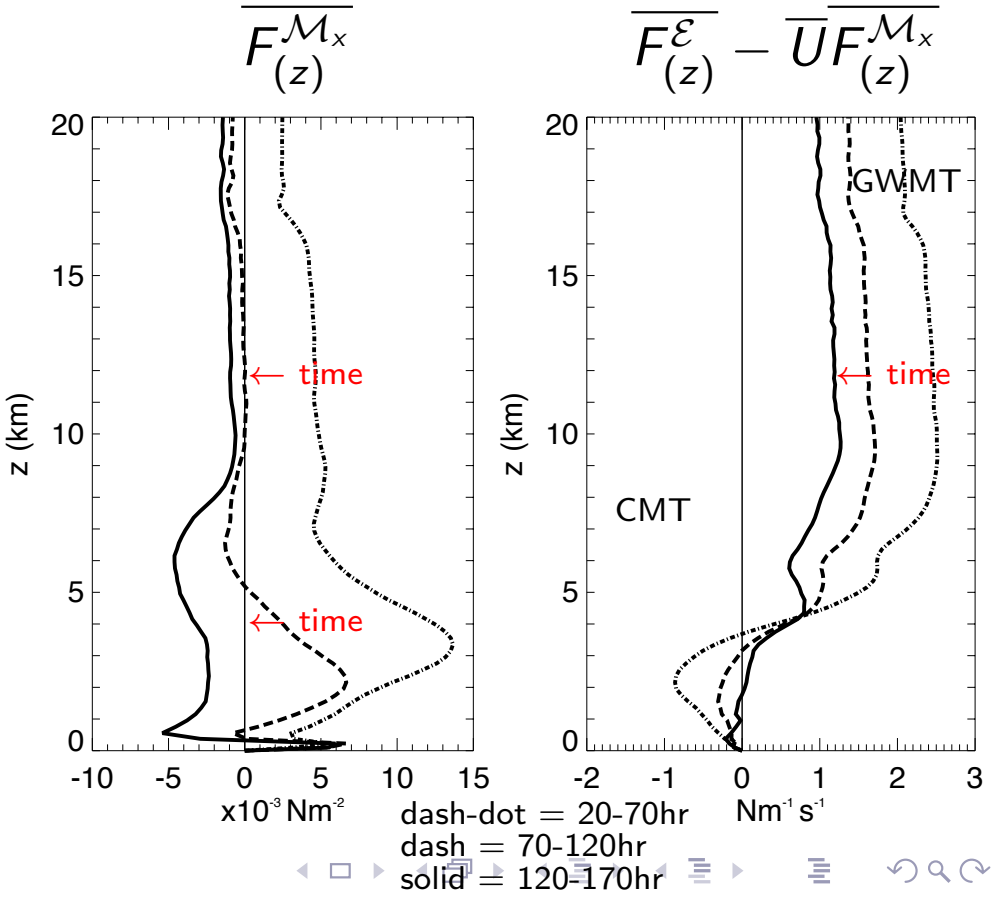


- ◇ Transient contribution peaks at ± 5 m/s
- ◇ Source/sink contribution peaks between 2-6 m/s

Momentum transfers in the presence of low-level shear

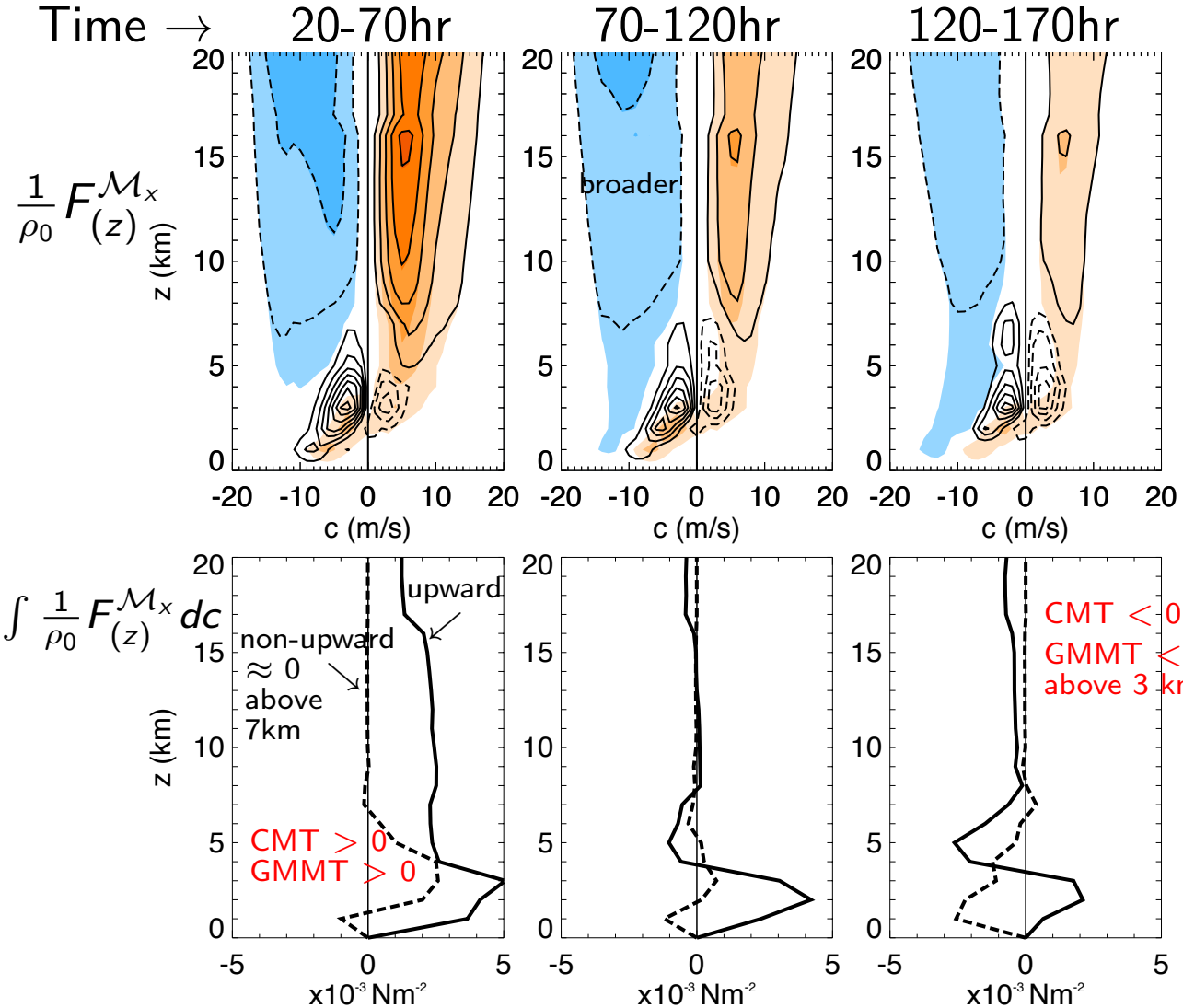
- In the presence of low level wind shear the convection becomes organized; transitions from upshear propagating (positive tilt) to quasi-stationary (upright tilt) and finally to cloud clusters (negative tilt).



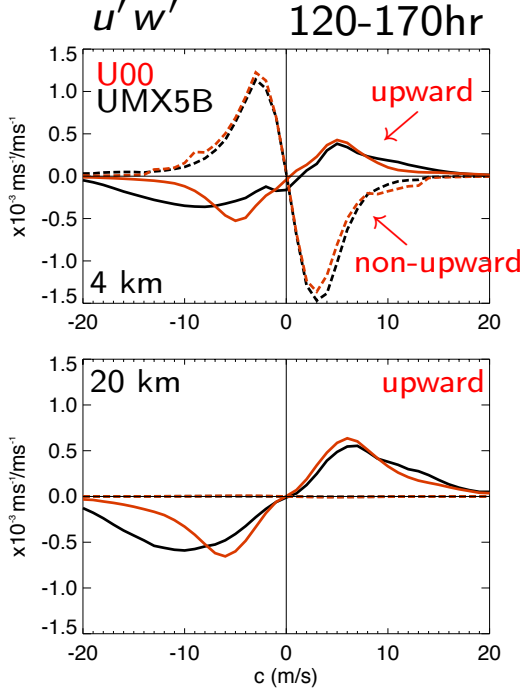


Momentum transfers in the presence of low-level shear

- Phase speed versus height co-spectra for UMX5B:



◇ Spectrum no longer symmetric

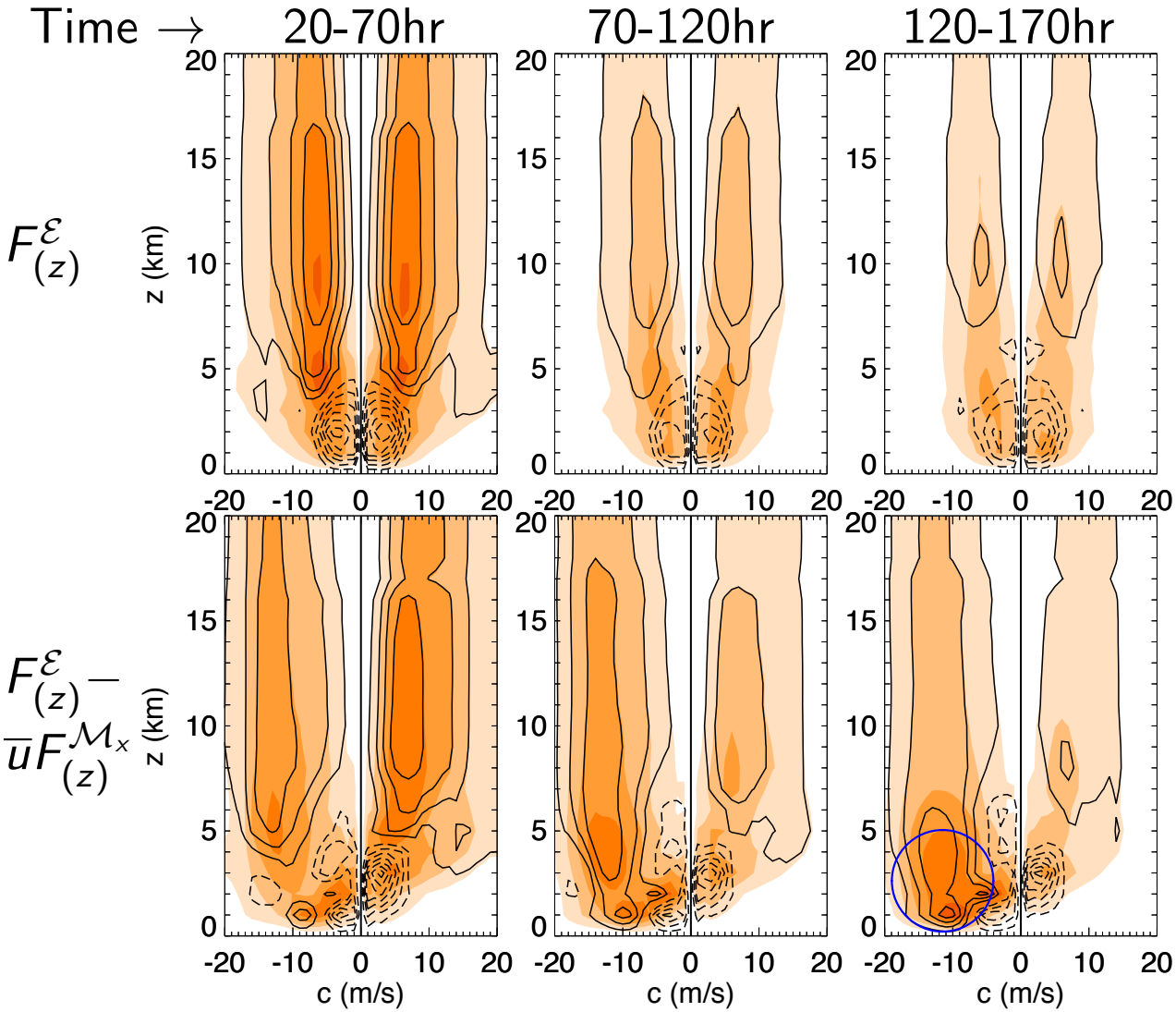


◇ Non-upward flux contribution is zero above 7km

◇ Upward flux contribution is large below 10km

Momentum transfers in the presence of low-level shear

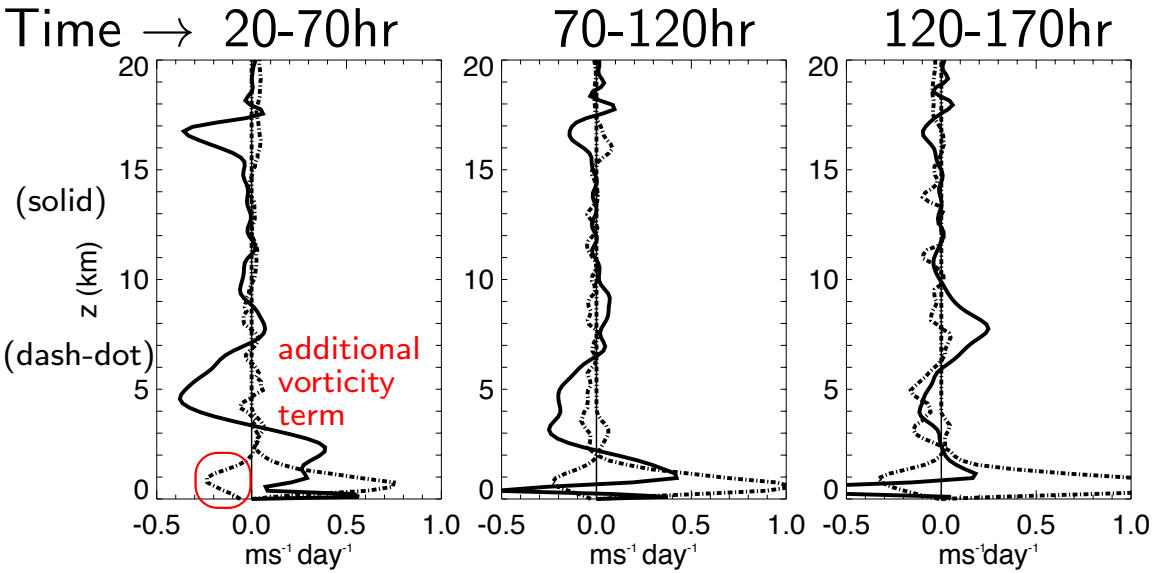
- Pseudoenergy flux co-spectra for U00 & UMX5B:

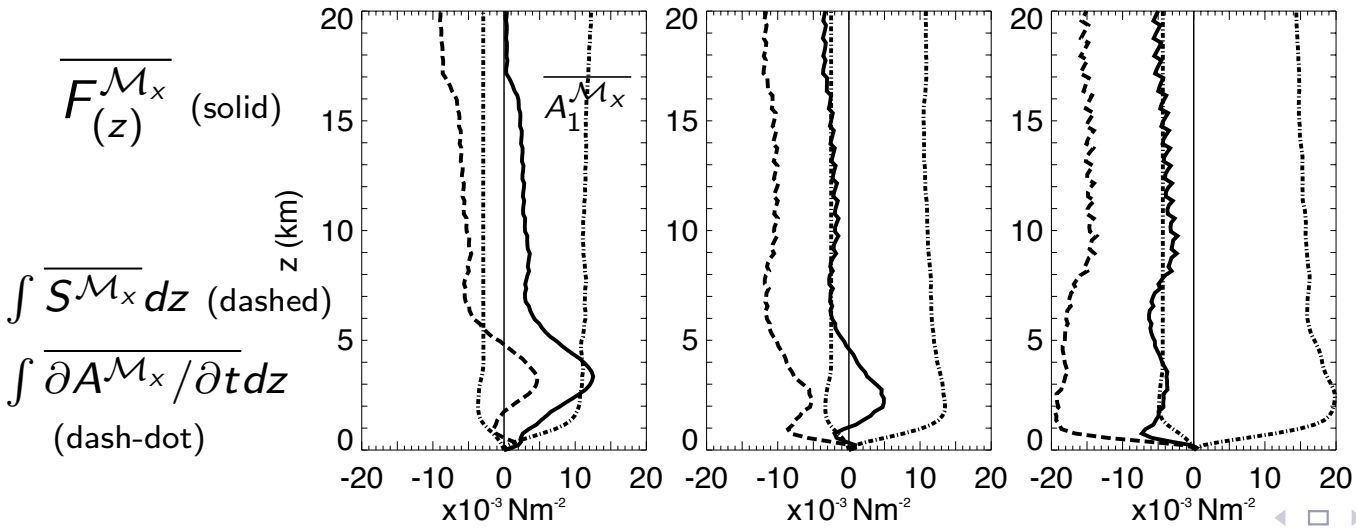


- ◇ Spectrum symmetric for U00
- ◇ Flux weakens with time
- ◇ Broadening of westward part of the spectrum
- ◇ Enhanced upward propagation between 0-5 km
- ◇ Peak between -5 to -15 m/s
- ◇ Slight enhancement of eastward part

Momentum transfers in the presence of low-level shear

- Pseudomomentum flux convergence and its contributions





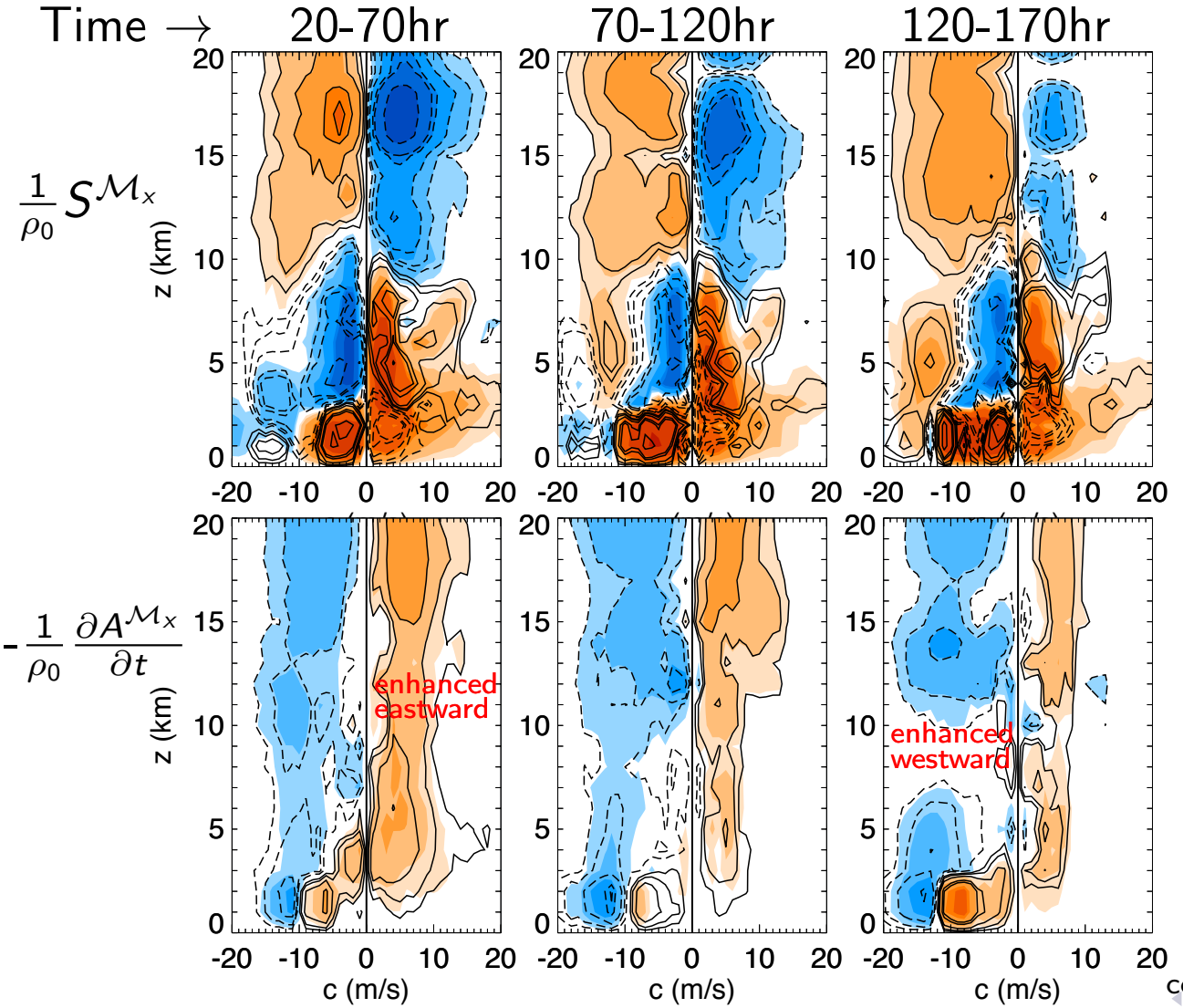
- ◇ Transfers are much stronger in the troposphere
- ◇ Large transfers near the surface

$$\frac{\partial \overline{F^{\mathcal{M}_x}_{(z)}}}{\partial z} = \overline{S^{\mathcal{M}_x}} - \overline{\frac{\partial A^{\mathcal{M}_x}}{\partial t}}$$

- ◇ Balance between $S^{\mathcal{M}_x}$ and $A_1^{\mathcal{M}_x}$ transience
- ◇ Flux $F^{\mathcal{M}_x}$ is on the order of $A_2^{\mathcal{M}_x}$ transience

Momentum transfers in the presence of low-level shear

- Phase speed versus height co-spectra for UMX5B:



- ◇ Divergence/convergence pattern consistent with a source between 3-10km and a sink above 10km
- ◇ The convergence for $c > 0$ below 5km is not consistent with upward propagation

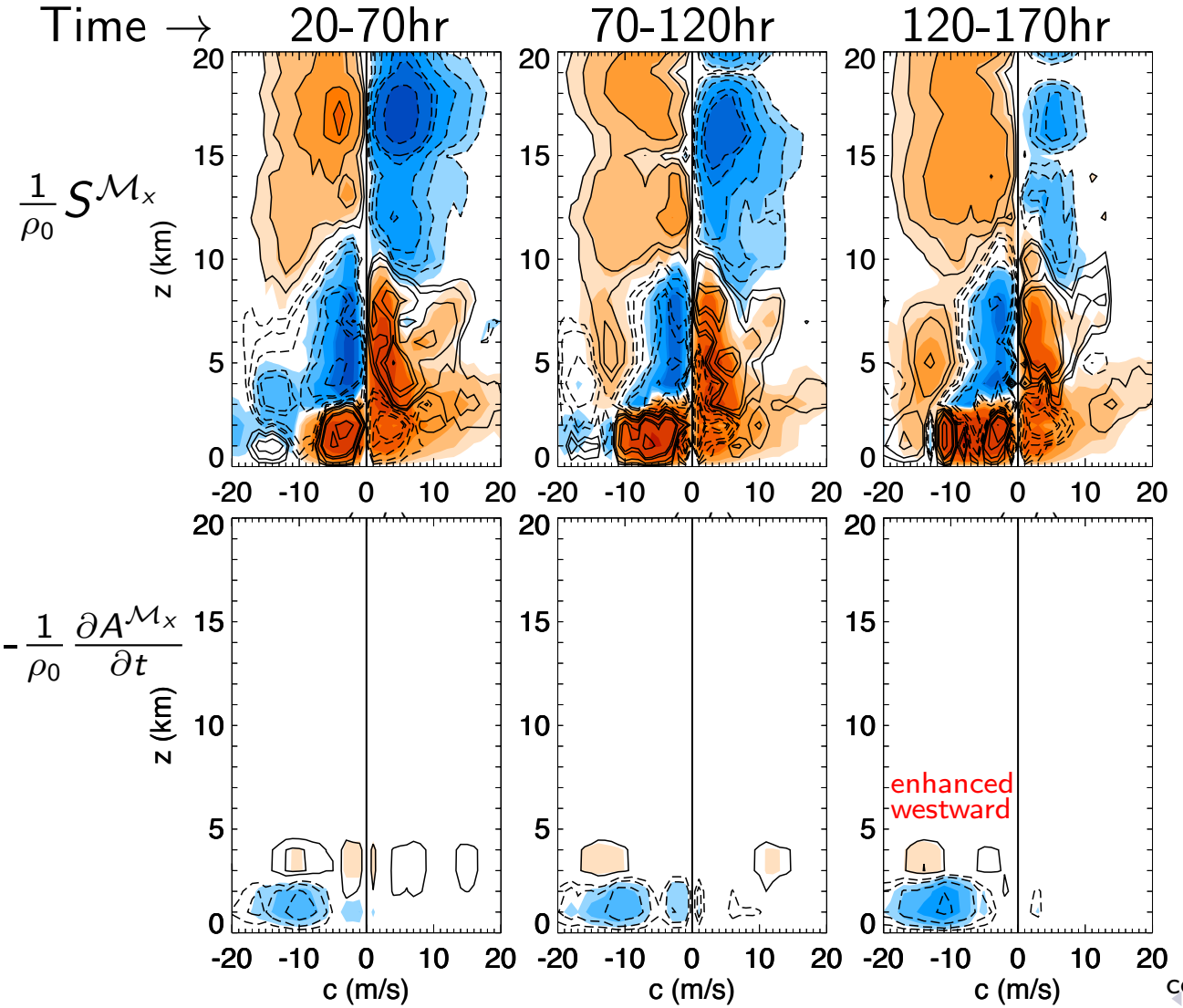
$$\overline{\frac{\partial F_{(z)}^{\mathcal{M}_x}}{\partial z}} = \overline{S^{\mathcal{M}_x}} - \overline{\frac{\partial A^{\mathcal{M}_x}}{\partial t}}$$

- ◇ Transience provides a source at all vertical levels
- ◇ Transience contribution consistent with upward propagation

contour = 0.025 m/s/day

Momentum transfers in the presence of low-level shear

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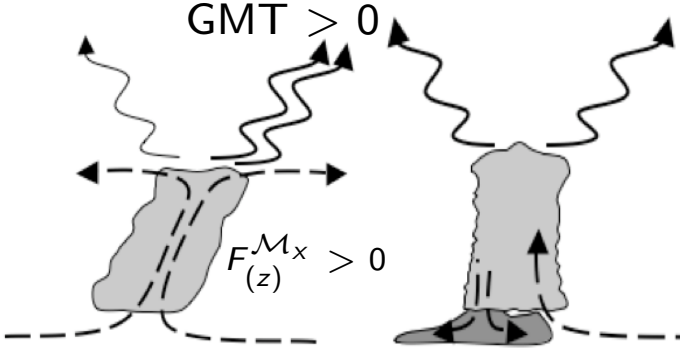
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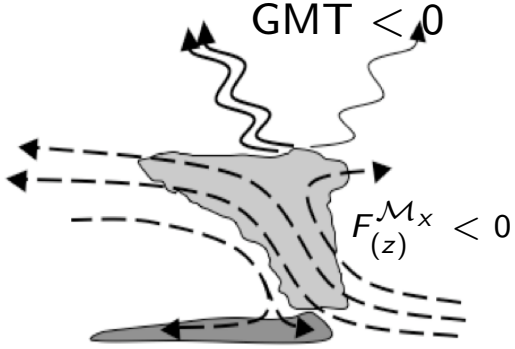
Momentum transfers in the presence of low-level shear

- CMT transitions from positive to negative.
  → Consistent with convective regimes (Houze 2004).
- GWMT shows similar transition (LM10).
- Both show a broadened westward spectrum between 10-20 m/s.
  → Transience is responsible for the broadened spectrum
  → Associated with transient updrafts in the organized convection.

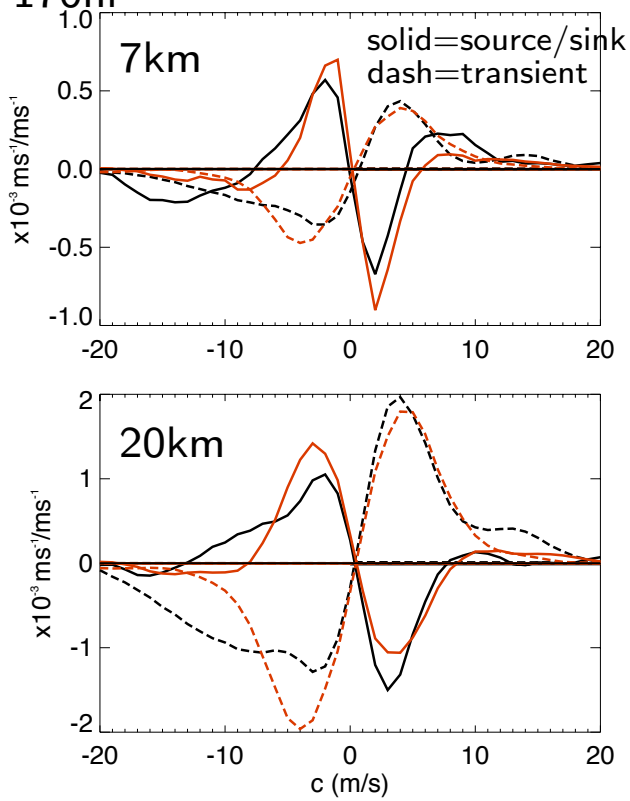
20-70hr: CMT > 0
GMT > 0



120-170hr: CMT < 0
GMT < 0



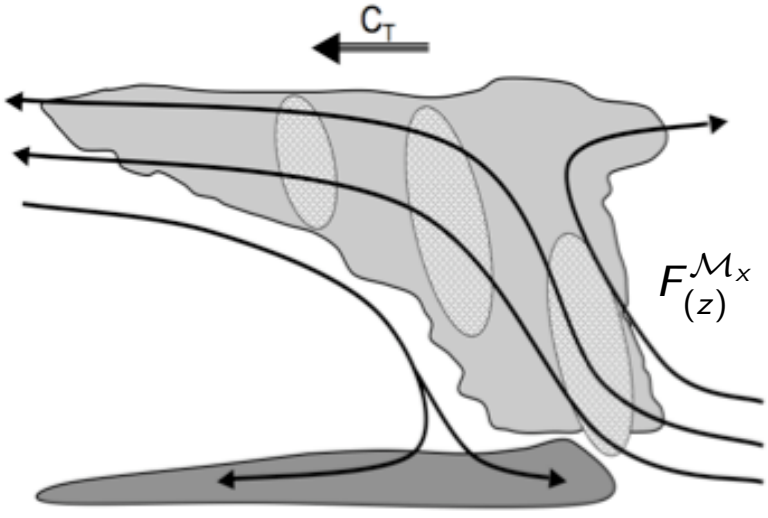
Contribution to spectra
120-170hr



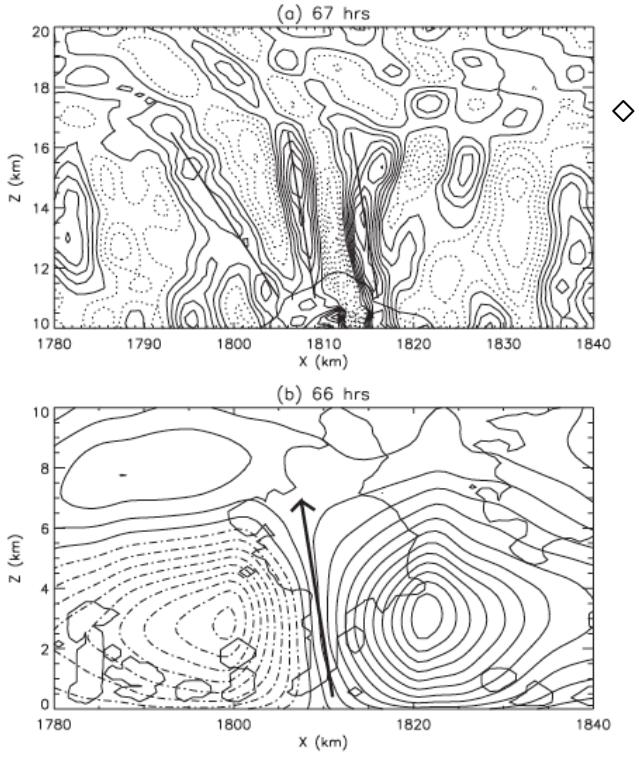
- ◇ Transient contribution has a much broader westward spectrum
- ◇ Source/sink contribution has a weaker broadening

Momentum transfers in the presence of low-level shear

- When the convection is organized transient updrafts move rearward in the cloud frame creating a transient signal.



$F_{(z)}^{\mathcal{M}_x} < 0$



◇ Transient updrafts move rearward relative to the core

◇ Spectral bias arises because tilted source projects onto waves propagating in the direction of tilt

→ Effect of transient updrafts noted by Fovell et al. (1992)

Conclusions

- Pseudomomentum wave-activity conservation law can be used to understand the mesoscale transfers of momentum associated with convection and gravity waves.
  - $\rightarrow$ CMT and GWMT contributions can be isolated using upward propagation criteria (first Eliassen-Palm theorem).
  - $\rightarrow$ Can be used to isolate CMT and GWMT and their sources and sinks in cloud resolving model simulations.
- Low-level shear significantly impacts the CMT and GWMT spectrum both above and below the tropopause.
  - $\rightarrow$ Leads to a broadened phase speed spectrum.
  - $\rightarrow$ Transience from the organized convection can be connected to signals above the tropopause and cannot be ignored.
- CMT and GWMT should not be treated independently in climate models.
  - $\rightarrow$ GWD parameterizations should account include transient sources and the effects in the source region.

Application to mesoscale momentum transfer

- Dispersion relation

$$(\sigma - Uk)^2 - \frac{kU_{zz}}{k^2 + m^2}(\sigma - Uk) - \frac{N^2 k^2}{k^2 + m^2} = 0$$

- Long wave limit, e.g. $k^2 + m^2 \ll U_{zz}/N^2$, vorticity waves:

$$\sigma \sim Uk + \frac{kU_{zz}}{k^2 + m^2}$$

- Short wave limit, e.g. $k^2 + m^2 \gg U_{zz}/N^2$, gravity waves:

$$\sigma \sim Uk + \frac{Nk}{(k^2 + m^2)^2}$$

Application to mesoscale momentum transfer

- Pseudoenergy

$$A^{\mathcal{E}} = |\mathbf{v}'|^2 + \frac{\rho_0}{2}\left[\frac{1}{(\overline{\theta}_z)^2} - \frac{\rho_0 \overline{u}}{(\overline{\theta}_z)^2}\left(\frac{U_z}{\rho_0}\right)_z\right](\theta'^2)^2 + \frac{\rho_0 \overline{u}}{\theta_z}\omega\theta'$$

with

$$F^{\mathcal{E}}_{(z)} = c_p \rho_0 \pi' w' + \rho_0 u' w'$$

Pseudomomentum wave-activity conservation law

- The 2D pseudomomentum $A^{\mathcal{M}_x}$ wave-activity conservation law for a $[\overline{u}(z), \overline{\theta}(z)]$ background state is:

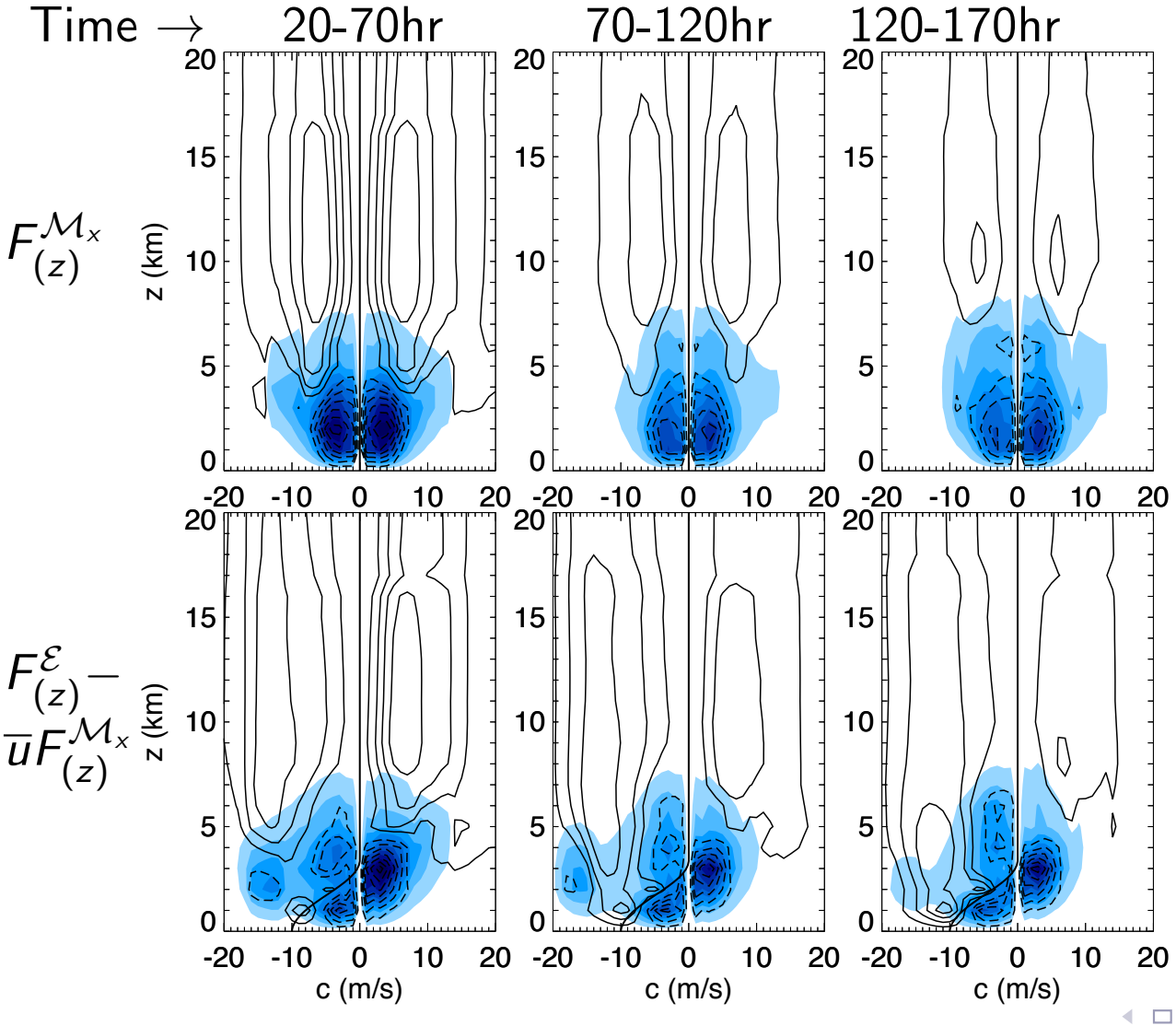
$$\frac{\partial A^{\mathcal{M}_x}}{\partial t} + \nabla \cdot \mathbf{F}^{\mathcal{M}_x} = 0$$

where

$$\begin{aligned} A^{\mathcal{M}_x} &= \frac{\rho_0}{\overline{\theta}_z}\omega'\theta' + (\overline{\omega} + \omega') \int_0^{\theta'} \left[R(\overline{\theta} + \eta) - R(\overline{\theta})\right] \, d\eta \\ &\quad -\rho_0 \int_0^{\theta'} \left[S(\overline{\theta} + \eta) - S(\overline{\theta})\right] \, d\eta \\ F^{\mathcal{M}_x}_{(z)} &= \rho_0 u' w' + w' A^{\mathcal{M}_x} \end{aligned}$$

Application to mesoscale momentum transfer

- Phase speed versus height co-spectra for UMX5B:



◇ Spectrum is symmetric

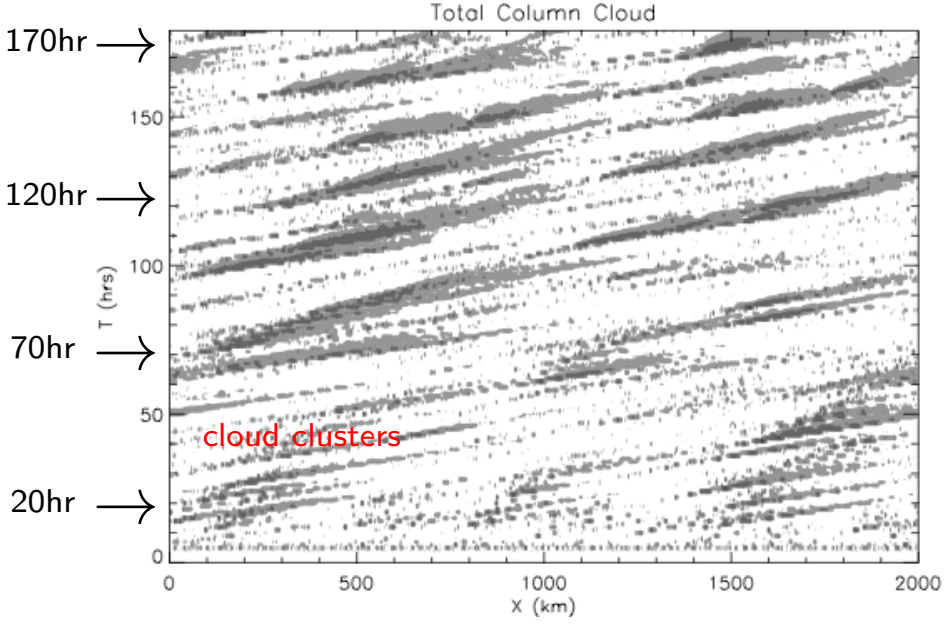
◇ Non-upward flux contribution is zero above 7km

◇ Upward flux contribution is large below 10km

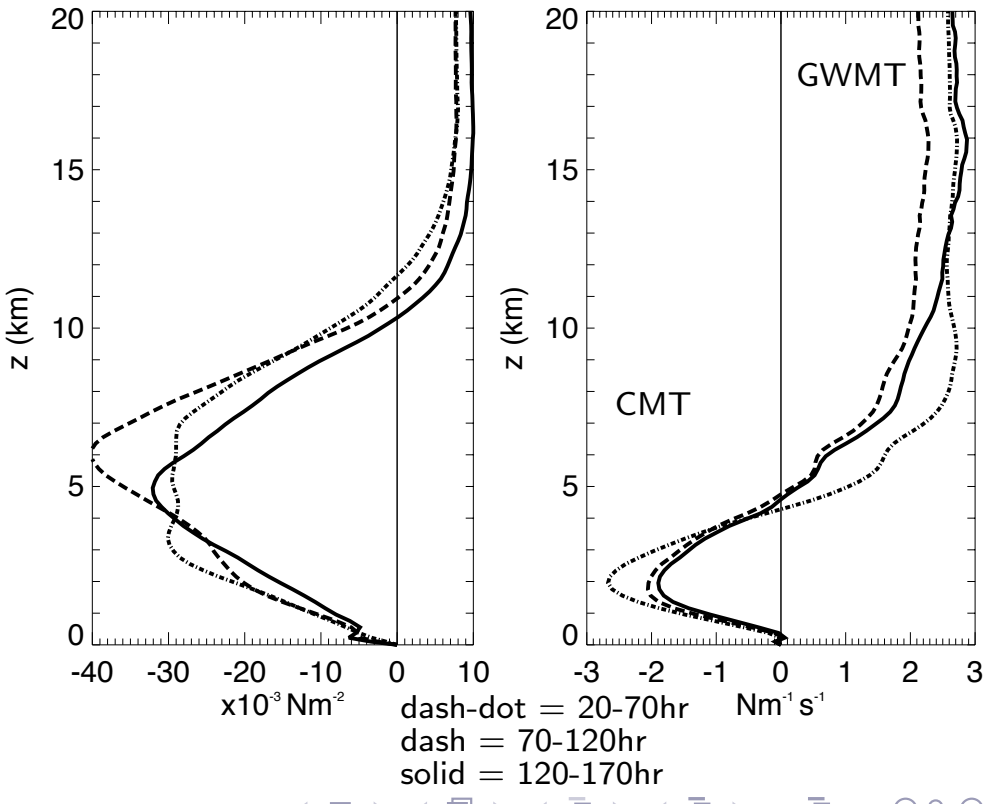
Application to mesoscale momentum transfer

- In the presence of mid-level wind shear the convection becomes organized very quickly; cloud clusters (negative tilt) emerge and propagate eastward.

Total cloud column

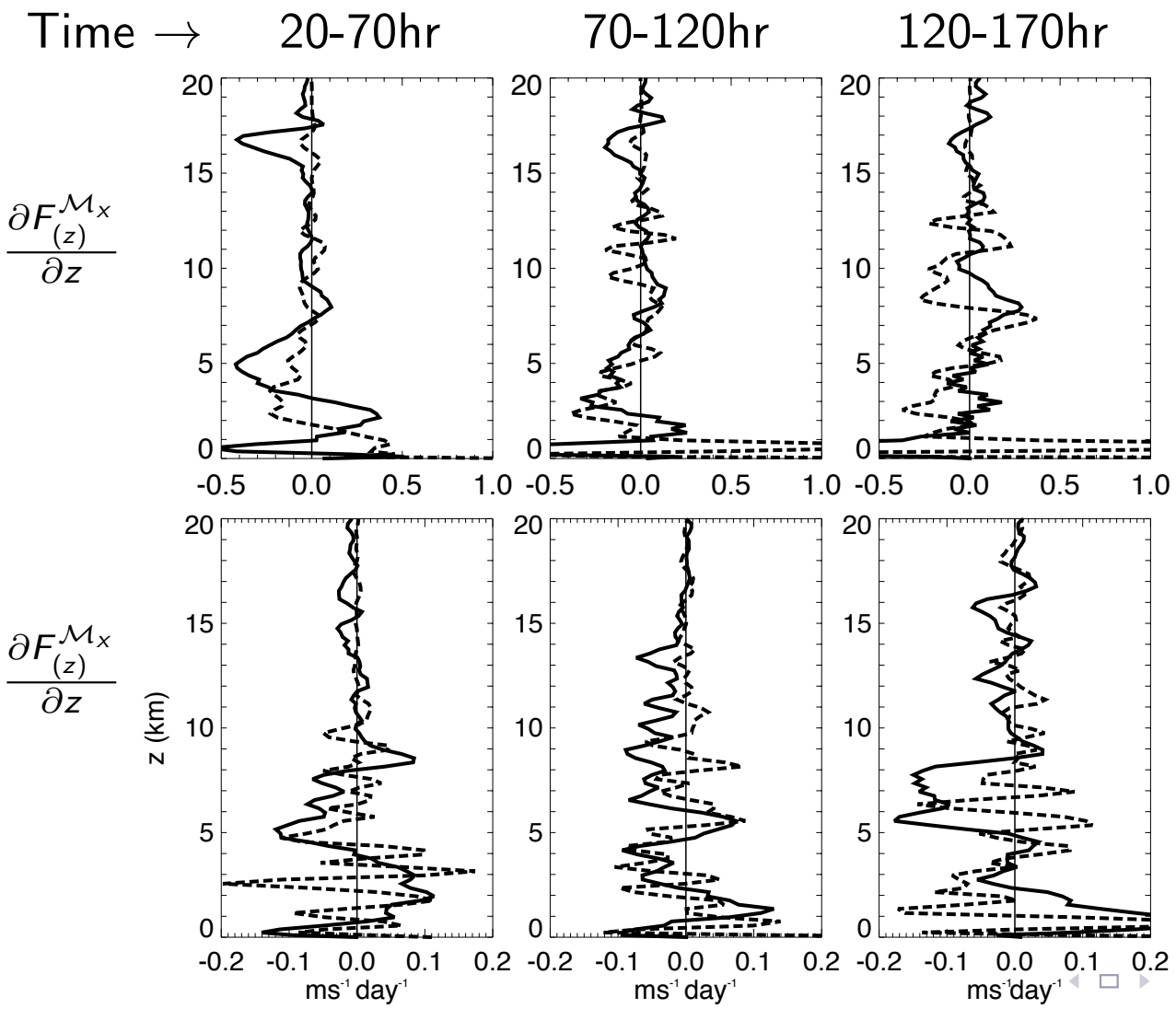


$\overline{F^{\mathcal{M}_x}_{(z)}}$ $\overline{F^{\mathcal{E}}_{(z)}} - \overline{U}\,\overline{F^{\mathcal{M}_x}_{(z)}}$



Application to mesoscale momentum transfer

- Contribution of non-linear flux term to domain-averaged profiles



Sources and sinks

- The source/sink terms in the wave-activity conservation laws involve the source/sink terms on the mesoscale

$$\begin{aligned}
\overline{S^{\mathcal{E}}} &= \overline{\mathbf{v}' \cdot S_{\mathbf{v}}} + \rho_0 \left[\frac{1}{(\theta_0)^2 N^2} - \frac{\rho_0 \mathbf{u}}{[(\theta_0)_z]^2} \cdot \left(\frac{\mathbf{u}_z}{\rho_0} \right)_z \right] \overline{\theta' S_\theta} \\
&\quad - \frac{\rho_0}{\Theta_z} \mathbf{u} \cdot \left[\overline{\boldsymbol{\omega}'^{\perp} S_\theta} - \overline{\theta' (\nabla \times S_{\mathbf{v}})^{\perp}} \right] \\
\overline{\mathbf{D}^{\mathcal{P}_{x,y}}} &= -\frac{(\rho_0)^2}{[(\theta_0)_z]^2} \left(\frac{\mathbf{u}}{\rho_0} \right)_z \overline{\theta' S_\theta} - \frac{\rho_0}{\Theta_z} \left[\overline{\boldsymbol{\omega}'^{\perp} S_\theta} - \overline{\theta' (\nabla \times S_{\mathbf{v}})^{\perp}} \right].
\end{aligned}$$

with

$$\overline{S^{\mathcal{E}}} = \overline{\mathbf{v}' \cdot S_{\mathbf{v}}} + \rho_0 \overline{\theta' S_\theta} / (\theta_0)^2 N^2 - \mathbf{u} \cdot \overline{\mathbf{D}^{\mathcal{P}_{x,y}}}$$

- The second term in $\overline{S^{\mathcal{E}}}$ is recognized as a term in the available potential energy budget since $\rho_0 \overline{\theta' S_\theta} / (\theta_0)^2 N^2 = \rho_0 \overline{\theta' w'} / \theta_0$.